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Revision of the version of 25 November 2008 in Høg (2008)
All links to my dropbox had become wrong in March 2017 but are now replaced by links to my website.

Astrometric accuracy during the past 2000 years

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ABSTRACT: The great development of astrometric accuracy since the observations by Hipparchus about 150 BC was documented in 2008 in the first version of the present report. This report is updated, e.g. with recent information on the catalogues before 1800 AD. The development has often been displayed in diagrams showing the accuracy versus time. A new such diagram is provided in a figure and in a .png file (Section 2) and this information will presumably be the main interest for some readers. For the specialist reader however a detailed documentation is provided in order to give confidence in the diagram and to show how our knowledge about astrometric accuracy has improved in the recent twenty years. The history of these diagrams is illustrated in the appendix.

1 Introduction

The present report shall document and discuss the accuracy of observed positions of stars. It replaces a previous report Høg (2008c) from 2008. The new report is available in two versions Høg (2017c) where the changes since 2008 stand red and Høg (2017d) which includes an appendix from 2008. The evolution of astrometric observations during the past centuries is shown in three tables of a report (Høg 2017b) to which I will refer: Table 1 for position catalogues, Table 2 for proper motion catalogues, and Table 3 for catalogues of trigonometric parallaxes.

The evolution has often been displayed in diagrams, showing the accuracy versus time. These diagrams have at least one thing in common, the improvement by many powers of ten from the half degree errors of Hipparchus, the Greek father of astronomy, to one milliarcsec (mas or millisecond of arc) for the diagrams including the Hipparcos Catalogue. But Tycho Brahe and Flamsteed are the only other sources always included, though with quite different numbers. Other differences are pointed out below. I will present a recommended diagram of astrometric accuracy, including explanations and a list of the sources, in literature or otherwise, for the points as plotted.

A detailed history of the various other diagrams is given in the appendix. These diagrams have puzzled me since 1985 and are therefore discussed here, showing the very significant differences. The diagrams used for the Hipparcos mission up to 1989 were based on a serious misunderstanding of a diagram from 1983. I constructed a more correct diagram in 1995 which was used in the Hipparcos book of 1997. An improved version was presented in 2008, showing the accuracy of positions and parallaxes in catalogues as based on included documention. The diagram was updated in 2016 as Figure 1a which has been included in, e.g., Høg (2016b).

Some of the diagrams give the impression of a smooth, gradual improvement over all the centuries, including the last 500 years. This obscures the historically interesting fact that jumps can be clearly seen in Figs. 1a and 1b from respectively 2016 and 2008. A *'jump'* means a *big improvement within a very short time* as the result of great investment of material resources and intellectual efforts.

The largest jumps have been obtained in modern times by space astrometry. The Hipparcos satellite launched by ESA in 1989 gave a factor 100 over the contemporary accuracy of positions obtained from the ground. The Gaia satellite mission from 2013 to about 2019 also by ESA is expected to yield another factor 100 over Hipparcos.

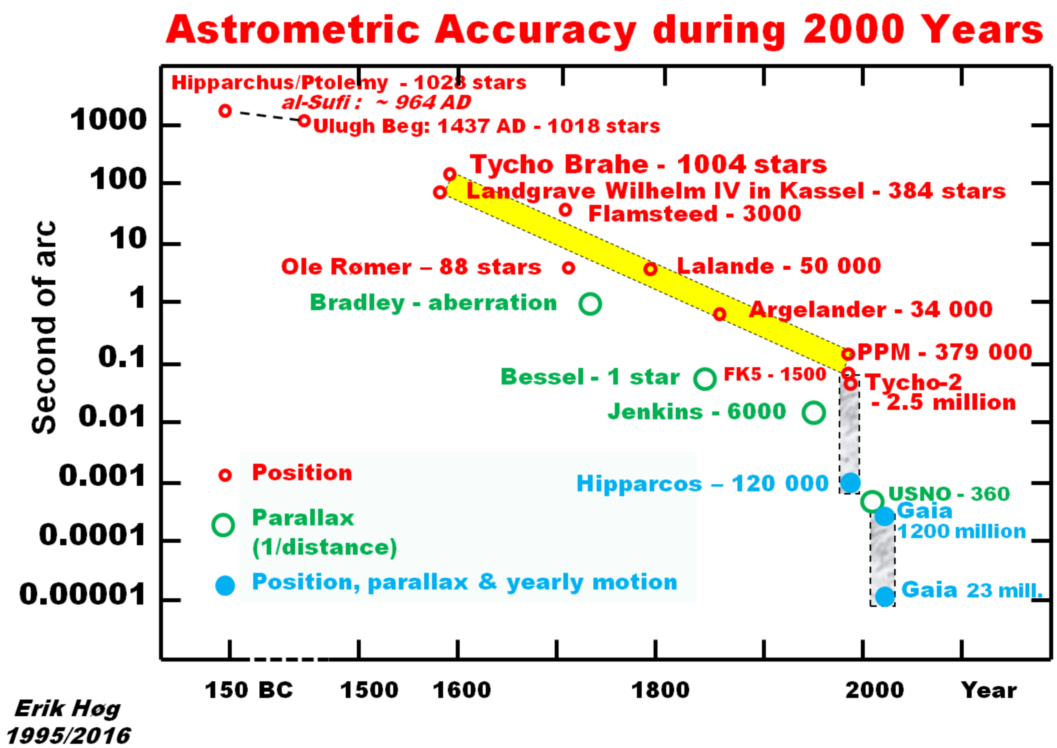

Fig. 1a. Astrometric accuracy during 2000 years: Høg-2016. The accuracy was greatly improved shortly before 1600 AD by the Landgrave in Kassel and by Tycho Brahe in Denmark. The following 400 years brought even larger but much more gradual improvement before space techniques with the Hipparcos satellite started a new era of astrometry.

A jump in accuracy was obtained more than 400 years ago by the Landgrave in Kassel and by Tycho Brahe before 1590 when they both measured positions ten times more accurately than Hipparchus/Ptolemy and Ulugh Beg. New information about the Landgrave has only reached me in 2015 although it was published by Jürgen Hamel first in 1998 and later in Hamel (2002). I am grateful to Professor Andreas Schrimpf for lending me that book at my visit to Marburg in November 2015.

The following sections contain: Section #2 the recommended diagram of astrometric accuracy, #3 the previous version of the diagram, #4 explanation to the diagrams, #5 sources for astrometric accuracy, #6-8 sources for Tables 1-3 in Høg (2017b), #9 explanation to the appendix, and #10 references.

2 Recommended diagram

Figure 1a is a diagram of the development of astrometric accuracy with time prepared in 2016 for the present report. It is called Høg-2016 since, for convenience, a diagram is designated by "name-year". It is available in the present report at a link to a .png-file at Høg (2016).

The points are placed at the mean observation epoch, except the compilation catalogues FK5, PPM, and Jenkins which are placed at the year of publication and with the accuracy of the positions in FK5 and PPM in that year. The circles refer to "positions" and "parallaxes", the word "best" from the older diagram has been omitted as being misleading because we want to show median values of the standard errors in each catalogue, representative for the bulk of stars in a catalogue. It has been suggested to include more information on the most accurate stars in each catalogue, but the diagram would be more complicated and it would be very difficult to collect the information and to present it well in a graph.

One of the points has been included for historical reasons but does not represent star positions: The Persian astronomer Abd ar-Rahman al-Sufi (or as-Sufi or Azophi) provided magnitudes of many stars in the catalogue of Ptolemy and he thus represents here the important muslim astronomers before Ulugh Beg. The points for ROEMER and SIM represented two important proposals for astrometry from space and were included in the 2008-diagram, but are omitted in 2016.

In 2017 I was informed about Wittmann (2012), a study of the accuracy obtained by Tobias Mayer about 1756, but no point has been added to the diagram because it is already rather overloaded. Further comments to the diagram are placed at the end of Sect. 4.

3 Previous versions of the diagram

The first version of my accuracy diagram is shown in the appendix as Høg-1995. It was drawn in 1995 in correspondence with several colleagues from the Hipparcos Science Team and appears as Fig.1 in Vol.1 of The Hipparcos and Tycho Catalogues. Two principles were followed in this diagram, but apparently not always in similar diagrams by other authors: It shows catalogue errors of single *stars* rather than errors of single *observations* and it only shows some of the most accurate catalogues of the given time. To be precise: I am plotting the *median external standard error per star in the catalogues, if available*. In most catalogues bright stars are more accurate than faint ones, but since only one number can be accommodated in the diagram, I find a median value most representative which then typically corresponds to *the error for the faint stars of a catalogue.*

Figure 1b shows the previous version of the diagram, called Høg-2008, prepared in 2008 for the report Høg (2008d). Changes in the diagram Høg-2008 compared with Høg-1995/2005 are: Hipparchus/Ptolemy 60' instead of Hipparchus 20', The Landgrave of Hesse is the correct English name instead of Hessen, Flamsteed 20" instead of 12", and 3000 stars instead of 4000, Lalande is now included, for Argelander a larger catalogue of 34000 stars at 0.9", PPM, FK5 and Tycho-2 slightly corrected, Roemer proposal 1992 is included because this proposal led to Gaia and the other astrometry satellite projects DIVA, FAME, and JASMINE. Gaia is here plotted with 1200 million instead of "many" million stars, and Gaia is shown with two dots in order to give more information. Bradley-aberration is included, USNO is updated to 360 stars instead of 100; the dot for SIM has been placed at 3 muas with 10,000 stars, although 1300 stars would be more correct at this accuracy, but space in the diagram is limited; see further explanations in the following section on sources.

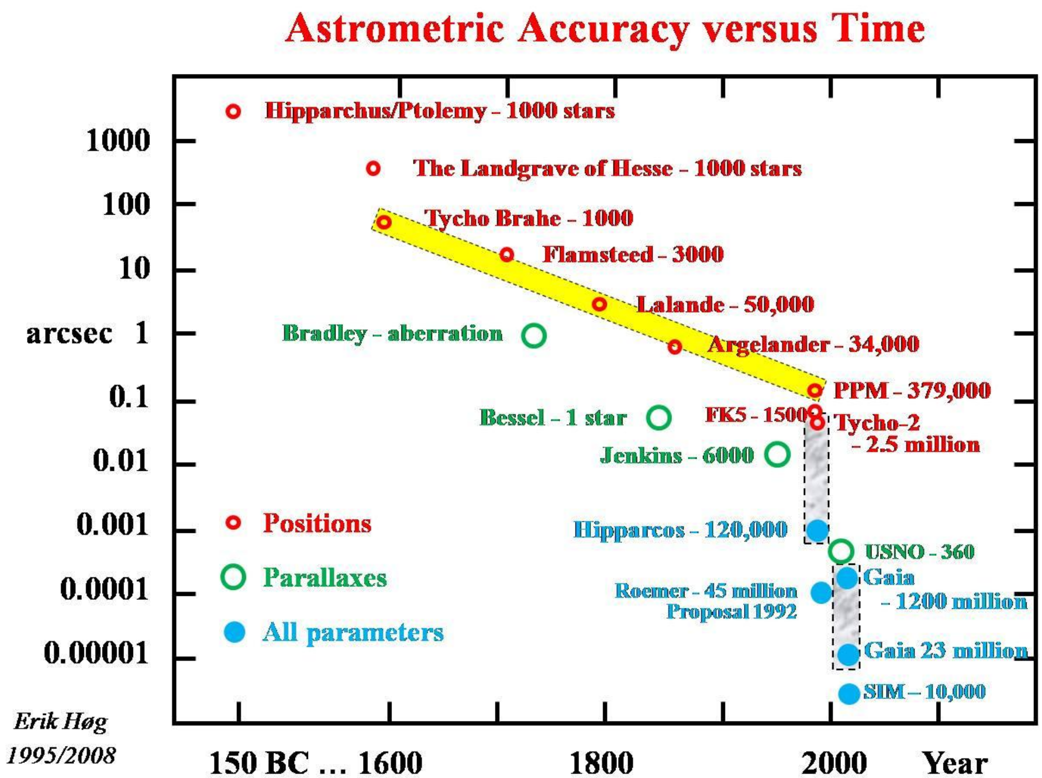

Figure 1b. Astrometric accuracy during 2000 years, Høg-2008.

4 Explanation to the diagrams

Astrometric accuracy during 2000 years

Errors of star position coordinates and parallaxes in accurate catalogues are shown in Figs. 1a. It appears that the Landgrave in Kassel and Tycho Brahe at the same time make a breakthrough to much better accuracy for positions of stars by obtaining ten times smaller errors than their predecessors. A gradual improvement by a factor 1000 follows during the next 400 years before the jump in accuracy by a factor of almost 10 000 is achieved by the two astrometric ESA satellites Hipparcos and Gaia. The diagram also shows the enormous increase of number of stars as well as the improvement for measurement of small angles and parallaxes.

Detailed explanation

Errors of star position coordinates and parallaxes in accurate catalogues are shown in Fig. 1a. This means the *median external standard error per star in a catalogue, if available*. In most catalogues bright stars are more accurate than faint ones The representative median error, dominated by faint stars, is given for most catalogues.

The diagram Høg-2008 in Fig. 1b was outdated in 2016 with respect to the time before 1800 AD. It had this text in 2008: "It appears that the Landgrave of Hesse was able to measure positions with errors about six minutes of arc, ten times better than Hipparchus/Ptolemy in the Antique. A few years after the Landgrave and thanks to generous support from the king of Denmark, Frederik II, Tycho Brahe reduced the errors by a further factor of six. The Landgrave and Tycho, both wanted to equal Hipparchus by reaching the same number of 1000 stars." This information was revised in 2016 and the values for Flamsteed and Lalande were improved by data from Lequeux (2014a) which contains an updated detailed version of our Fig. 1b.

In the new diagram Høg-2016 in Fig. 1a it appears that the Landgrave in Kassel was able to measure positions with errors about 1.1 minute of arc, forty times better than Hipparchus/Ptolemy in the Antique. At the same time as the Landgrave and thanks to generous support from the king of Denmark, Frederik II, Tycho Brahe also obtained an accuracy about 1 arcmin. His large catalogue of 1000 stars had about 2 arcmin errors. Tycho Brahe reached the same number of 1000 stars as Hipparchus, while the Landgrave did not have this aim and observed almost 400 stars.

A period of 400 years followed with gradual improvement of the accuracy as astronomers always made use of the best technical possibilities of their time, especially with better time-keeping equipment and accurate manufacturing of mechanics, optics, and with electronics. The accuracy was improved by a factor about 1000 in 400 years, i.e. a factor 5.5 per century, and the number of stars was greatly increased.

The introduction of space techniques, however, with the Hipparcos mission gave a veritable jump in accuracy by a factor of 100 with respect to FK5, the most accurate ground-based catalogue ever. Hipparcos obtained a median accuracy of 0.001 arcsec for positions, annual proper motions and parallaxes of 120 thousand stars. The positions in the Tycho-2 Catalogue also from the Hipparcos mission but with 2.5 million stars are as accurate as the positions in FK5 which contains only 1500 bright stars. Tycho-2 includes annual proper motions, derived from Tycho-2 positions and more than 140 ground-based position catalogues, but no parallaxes. The median standard error for positions of all stars in Tycho-2 at the observation epoch 1991 is 60 mas, and it is 7 mas for stars brighter than 9 mag. The median error of all proper motions is 2.5 mas/yr.

The points marked “parallaxes” might be labelled “small-angle astrometry” or “relative astrometry”, and all ground-based measurements of parallaxes are of that kind. This is about ten times more accurate than large-angle astrometry which was required to measure the positions shown in the diagram. The first such point is “Bradley – aberration” shown at 1.0 arcsec, the accuracy which Bradley obtained for the constant of aberration with his zenith sector. The accuracy of ground-based parallaxes begins with Bessel’s single star in 1838, followed by a factor 100 improvement in accuracy at the U.S. Naval Observatory in Flagstaff since about 1990 for faint stars.

“All parameters” means that about the same accuracy is obtained for *annual* proper motions, positions and parallaxes, as was in fact achieved with Hipparcos for the first time in the history of astronomy. The Roemer proposal of 1992 (Høg 1993) introduced CCDs in integrating scanning mode in a space mission, instead of photoelectric detectors as in Hipparcos. Roemer promised a factor 10 better accuracy than Hipparcos for many more stars, and this proposal started a development of the design which led to the Gaia mission which was launched in December 2013. For Gaia an improvement by a factor of 100 over Hipparcos was predicted before launch for the 23 million stars brighter than 14 mag, i.e. 10 microarcsec median error. The median accuracy is expected to be 180 microarcsec for the 1200 million stars in the Gaia catalogue brighter than 20 mag, much better than the accuracy of Hipparcos. The two dots for Gaia thus show the expected accuracy for bright and faint stars.

Finally, in view of the expected Gaia results, studies are due about the scientific goals for ground-based optical astrometry after Gaia. Such a study is provided in Høg (2015) and briefly in Høg (2016e).

Further comments to Fig. 1a have been received from R. van Gent in July 2017, but no changes will be made at the present. Van Gent wrote:
1) From Hipparchus to the late 16th century, one would expect to see a horizontal line reflecting the precision of the Almagest catalogue – you could add a slightly lower horizontal line for Ulugh Beg's star catalogue as it was available for Muslim astronomers.
2) from the late 16th century until ca. 1725 one would expect to see a (lower) horizontal line reflecting the precision of Tycho's star catalogue – you could again also add a slightly lower horizontal line for the Hevelius star catalogue.
3) After 1725 you can have a downwards sloping line until the PPM catalogue illustrating the gradual increase in precision during the interval 1725 – 1989.

5 Sources for astrometric accuracy

Here follow the sources and reasoning for the accuracies used in the diagram of astrometric accuracy, and for Tables 1, 2 and 3 in Høg (2017b) "Selected Astrometric Catalogues", where the references are found if they are not included in the present report.

The standard errors

"Internal errors" of observations are obtained by analysis of repeated observations of the same stars at different times, as is usually done in meridian observation catalogues, e.g. in case of USNO (1920) from the n=10 observations. I have then derived the error for Table 1 by division with sqrt(10) because nothing else is available, but this "internal catalogue error" is not given in the catalogue, and it is certainly too small because of the unknown systematic errors.

The three tables of catalogues should ideally contain the "external errors" of a catalogue entry as would be obtained from a comparison with a more accurate catalogue. Such comparison has been carried out for nearly all catalogues from before 1800 AD using the Hipparcos catalogue. Beginning in the year 2010, Verbunt & van Gent have published studies of Ptolemy, Ulugh Beg, Tycho Brahe, and Hevelius and Lequeux (2014a) studied the 18th century. Mignard & Froeschlé (2000) studied the FK5, and I have used this comparison to derive below that the errors given in the FK5 of positions at the mean epoch and of the proper motions should be multiplied by a factor about 1.6 to obtain external errors.

For a historical catalogue it could be sufficient to take a representative sample of stars for the comparison as done by Lequeux (2014a). Most interesting in 2008 were the following catalogues where the errors in Table 1 might be wrong by a factor two: First priority had Flamsteed, and Lalande and Lequeux (2014a) has answered that question. A thorough comparison of Bradley/Auwers with Hipparcos by Brosche & Schwan (2007) is mentioned below.

In some cases reliable external errors have been derived, e.g. for the Hipparcos and Tycho-2 catalogues in the publications, and for the parallaxes in Jenkins' catalogue by Hertzsprung (1952). In case of Perth70 it is also believed that a reliable external error is known, as explained below at Perth70. The distinction between external and internal errors of catalogues is important for detailed comparisons, but it is difficult in many cases, if at all possible, to find sufficient information about this matter, and it cannot easily be presented in one line of a table. Internal errors are sometimes placed in brackets.

The standard errors in the tables are sufficient for the original purpose, to show the pace of development of astrometric accuracy over very long periods of time. But much care is needed in comparing within short intervals. I have below in some detail compared four meridian circle catalogues from within one century, i.e. two USNO catalogues from observations around 1907 and 1945 are compared with each other, with the Perth70 catalogue observed about 1970, and with the CMC1-11 catalogues observed about 1991.

It appears that the progress in accuracy and efficiency of meridian circles is rather modest in the first half of the 20th century where visual techniques were used, but the progress is very large in the second half thanks to photoelectric techniques and automatic control of micrometer and telescope. This large progress is independent of the Hipparcos mission, but the further progress thanks to the Tycho-2 Catalogue and recording with CCDs in meridian circles is truly tremendous.

The “accuracy of catalogued star positions” is the title of section 3.2.4 in Eichhorn (1974). He discusses theory and practice of this matter in the past where the available means of computation called for simple methods, and in his own time where electronic computers had made rigorous numerical methods feasible. In section 2.2.8 Eichhorn discusses “the accidental accuracy of relative visual positions”. He includes three tables adapted from Cohn (1907b), not (1970) as a typo has produced. Some trivial mistakes both in Eichhorn's extract and in Cohn's original paper make the use of the tables cumbersome, as I discuss below at Bradley/Auwers. This is meant as a call for caution.

Catalogues before 1700 AD

The following description of catalogues contains the new information about the old catalogues as well as the now outdated information from the 2008 version of this report. The accuracy of an observation catalogue of positions is plotted at the mean epoch.

Ptolemy, Ulugh Beg, Wilhelm IV, Tycho Brahe, and Hevelius
The catalogues with 1000 stars from Ptolemy, Ulugh Beg and Tycho Brahe were the largest up to Hevelius' catalogue of 1690. The catalogue from Kassel contains 384 stars. They were all observed with non-telescopic instruments.

Ptolemy and Ulugh Beg
Ptolemy's catalogue is the only extant from antiquity, and perhaps partly copied from the 300 years older Hipparchus. Ulugh Beg published his catalogue in 1437 in Samarkand, but it became known in Europe only when published again in 1665. The accuracy of these catalogues was evaluated by Verbunt & van Gent (2012).

Verbunt & van Gent (2012) compare the positions in these catalogues with the positions obtained from the Hipparcos positions and proper motions, these positions have negligible errors even projected back 2000 years. The standard error for ecliptic longitude is about 20% larger than for latitude. A Gaussian fit gives errors of 25' for Ptolemy and 20' for Ulugh Beg if only stars with deviations less that 50' are included. When stars up to 100' are included the errors become 10-15% larger. This shows that some stars in the two catalogues lie far from real stars on the sky, about 20 stars of Ptolemy and one or two of Ulugh Beg are further away than 150'. Only one star of Ptolemy and three of Ulugh Beg cannot be identified with any real star. These numbers bear witness to the excellent quality of the two catalogues.

The Landgrave in Kassel

The catalogue of 384 stars observed by Christoph Rothmann and Jost Bürgi at the observatory of the Landgrave Wilhelm IV in Kassel was written in its final form already in 1587. It was known to colleagues but it was only published in 1666 as appendix to Tycho Brahe's observation journals at a time so late that it had no impact on astronomy. With 384 stars the goal of 400 stars was reached, according to Hamel (2002) it was never planned in Kassel to reach the 1000 stars of Ptolemy as has often been claimed, e.g. by Eichhorn (1974).

The quality of the catalogue was first published by Hamel (1998, 2002). He studied the accuracy of positions by comparison with the SAO catalogue which contains proper motions with sufficient accuracy to project the positions 400 years back in time. For 361 safely identified stars out of the 384, the accuracy is 1.4' on average, 1.2' in right ascension and 1.5' in declination. The systematic errors are respectively 0.22' and 0.82'. In addition, a systematic error of 6' is found for the ecliptic longitude which has been explained by the use of the much too small distance to the Sun found in Antiquity.

Pannekoek (1961) writes very positively about the Landgrave without any numbers. He writes: "from Cassel came the first West-European catalogue of stars based on new measurements", and a letter from the Landgrave to king Frederik II about Tycho Brahe after his visit to Kassel in 1575 is mentioned as very important. The accuracy given by Eichhorn (1974) as 6' rms about a catalogue from 1594 with 1004 stars is incorrect, see Table 1b, his sources of information are not given and I have not been able to find them.

Added in 2016:
During a visit to Marburg in November 2015 I was informed about the study by Hamel. I have written a detailed article in Danish (Høg 2016b) and shorter ones in English (Høg 2016c, d) about the Landgrave and Tycho where I emphasize that the Landgrave and his team in Kassel deserve more credit for development of new instruments, for observations with high accuracy and for the interaction with Tycho Brahe. Between 1560 and 1567 Wilhelm himself observed a catalogue of 58 stars with an accuracy of 2.2' in right ascension and 4.6' in declination, as we now know from Hamel, i.e. an error 4-6 times smaller than reached by Ulugh Beg. Thus a breakthrough to higher accuracy was reached in Kassel already in 1567.

The large Kassel catalogue has recently been studied by Schrimpf (2016, priv comm. in April) who has kindly checked the following quite detailed extract from his email. These details show the extremely careful work of Christoph Rothmann in producing the catalogue. The high accuracy found by Hamel is confirmed and more than that. Schrimpf used the original handwritten manuscript as also Hamel did. He found that the later printed editions have errors: The Curtz edition has 43 typing errors, the Flamsteed edition 37, thus the original catalogue is much better than the Curtz or Flamsteed editions! By using the equinox 1586 and the same obliquity as Rothmann, Schrimpf obtains an offset in RA of 6.2'.

Neglecting all objects with positional errors larger than 10', he could use 376 of 384 stars. The catalogue contains 387 entries but 3 of them are double entries of the same star. Of the 384 distinct objects two are nebulous, the Messier-objects M44 and NGC 869. The position of the brightest star in each cluster is used in the analysis of the accuracy of the catalogue by Schrimpf. Ecliptic coordinates are given for these 384 stars and equatorial coordinates for 343 of them thus allowing valuable checks. It appears to Schrimpf that the positions have probably been corrected for refraction by Rothmann.

The new analysis of the listed equatorial coordinates gives standard errors of 1.17' and 1.11' for respectively RA[1] and Dec and average differences of 0.03' and 0.76'. The standard errors are computed from the deviations (*d*) for the 376 among all 384 Kassel stars within a circle of 10' radius centered on the Hipparcos position. Only 4 of these stars lie more than $3\sigma_d$=4.8" away, the distribution of deviations is slightly asymmetric. A systematic deviation of declinations is found south of -20 deg, up to +4' at -30 deg the southern limit of the catalogue with an altitude of 9 deg in the meridian, probably due to refraction.

A similar analysis of the ecliptic coordinates also given in the catalogue shows a systematic error, a correlation of latitude as function of longitude with an amplitude of 1.84 arcmin and a period of 360 degrees. The correlation disappears however when using the equinox of 1586 and the same obliquity as Rothmann (which is 1.91 arcmin smaller than the modern mean obliquity) when he computed the ecliptic coordinates from the originally measured equatorials, see Granada et al. (2003).

The accuracies of 1.17' and 1.10' in RA and Dec respectively for the 376 stars agree reasonably with Hamel's accuracies of 1.2' and 1.5' respectively for 361 stars; the larger value in Dec may partly be due to the use of the SAO catalogue instead of Hipparcos. It is impressive that only (384-376/384)=2% of the stars had deviations larger than 10', compared with 15% in Tycho's catalogue.

A. Schrimpf intends of course to complete and publish his findings. He will perhaps compute the accuracy of the positions for the 376 Kassel stars as observed by Tycho Brahe.

Tycho Brahe

Tycho Brahe's catalogue of 1004 stars from 1598 exists in a version by Tycho himself and one by Kepler printed in 1627. The accuracy of Kepler's version has been investigated most deeply by Verbunt & van Gent (2010a) comparing with the Hipparcos catalogue. They find good agreement with studies by Dreyer in 1916 and Rawlins in 1993 and they conclude that the error distributions in ecliptic longitude and latitude have widths, standard errors, of 2', but there are more stars with large deviations than in a Gauss distribution. There are about 15% with deviations in excess of 10' which is attributed to error of computation or writing.

The accuracy for the 777 stars in Tycho's short catalogue, a part of *Astronomiae Instauratae Progymnasmata,* from 1602 would be interesting to know since the remaining 227 up to the 1004 may be less accurate because they were added rather late in order to equal the size of the Ptolemy catalogue.

The accuracy has also been studied by Hamel (1998) which is not mentioned by Verbunt & van Gent. Hamel finds standard errors in right ascension and declination of respectively 2.3' and 2.4' for the 794 safely identified stars in Tycho's catalogue, in fair agreement with the above 2' which I accept for Table 1 and the diagram. Tycho's nine standard stars have an accuracy of about 30 arcsec, according to Wesley (1978). The accuracy of 1' given earlier by me will still be true for many of the stars. It is therefore fair to say that the Landgrave in Kassel and Tycho Brahe at the same time reached an accuracy about one arcmin, 20 times smaller than their predecessors.

Pannekoek (1961) writes about Tycho Brahe that a comparison with modern values shows a mean error about 1'. Eichhorn (1974) does not give an accuracy for Tycho Brahe, but writes: "His star positions are less accurate than his measurements of planetary positions".

[1] "Standard error in RA" always means $\sigma_\alpha \cos\delta$

Hevelius

The catalogue by Johannes Hevelius with the positions and magnitudes of 1564 entries was published by his wife Elisabeth Koopman in 1690. Verbunt & van Gent (2010b) discuss its accuracy on the basis of comparison with data from the Hipparcos Catalogue. We quote here from the abstract, but the result is not plotted in our diagram for lack of space. They compare their results with an earlier analysis by Rybka from 1984, finding good overall agreement. The magnitudes given by Hevelius correlate well with modern values. The accuracy of position measurements is similar to that of Brahe, with σ = 2' for longitudes and latitudes, but with more errors >5' than expected for a Gaussian distribution. The position accuracy decreases slowly with magnitude. The fraction of stars with position errors larger than a degree is 1.5%, rather smaller than the fraction of 5% in the star catalogue of Brahe. - The accuracy value 20”, six times too small, was adopted for Table 1 in 2008 for reasons given below.

The instruments in Col. 2 of Table 1 and 1b: Abbreviations: *AQua* = azimuth quadrant, introduced in Europe by the Landgrave Wilhelm IV in Kassel in 1558, *Sex* = sextant, invented by Tycho Brahe in 1570.

Table 1 Position catalogues before 1700 AD as known in 2016. The standard error of the mean positions in a catalogue is meant to be a median value.

Catalogue	Instrument	Publ. *year*	Mean epoch *year*	Obs. period *years*	N *entries*	s_{star} s.e. of star *' or "*
Ptolemy	-	c.150	138		1028	25'
Ulugh Beg	-	c. 1437	1437	17	1018	20'
Wilhelm in Kassel	AQua, Sex	1587	1586	3	384	1.14'
Tycho Brahe	Many	1598	1586	20	1004	2.0'
Hevelius	Quad+Sext	1690	1670	20?	1564	2.0'

The following indented text and table about the period from Hipparchus/Ptolemy to Hevelius were written in **2008** and used for Table 1b and Fig. 1b. They were outdated in 2015 by new information as explained above.

Table 1b From the now **obsolete** Table 1 of 2008: Position catalogues before 1700 AD. The instruments used by Ptolemy and Ulugh Beg were listed in 2008 as *Sextant* which is certainly incorrect because this instrument was invented by Tycho Brahe in 1570. The number of stars in Wilhelm's catalogue is 383, not 1004 as often told in the literature.

Catalogue	Instrument	Publ. *year*	Mean epoch *year*	Obs. period *years*	N *entries*	s_{star} s.e. of star *arcsec*
Ptolemy	Sextant	150	138		1025	1 deg.
Ulugh Beg	Sextant	1665	1437	17	1018	1 deg.
Wilhelm of Hesse	Quad	1594			1004	360"
Tycho Brahe	Many	1598	1586	20	1005	60"
Hevelius	Quad+Sext	1690	1670	20?	1564	20"

Hipparchus/Ptolemy 1 degree at 150 BC
Ulugh Beg also obtained 1 degree accuracy in 1437, but he is not represented in the diagram. The catalogue in the Almagest by Ptolemy is the oldest extant star catalogue. It has been proposed that this catalogue is identical with that of Hipparchus, but this is not supported by Shevchenko (1990). The catalogues of Ptolemy and Ulugh Beg are nearly equivalent in merit, according to Shevchenko. They both have overall systematic longitude errors about one degree, and the systematic error has a scatter about one degree. The root-mean-square errors of the positions of the zodiacal stars in the two catalogues are about 20 arcminutes=1200”=0.33 deg, i.e. within constellations. Shevchenko explains the analogies as due to the fact that the Samarkand astronomers used the equipment and methods described in Almagest.

Eichhorn (1974) p. 101, says that the rms. errors of ecliptic latitudes and longitudes in Ptolemy's catalogue are 0.58 and 0.37 degrees, respectively, but I will stick to Shevchenko.

For the diagrams we have hitherto always shown Hipparchus with 1200”. This is really a local internal error within constellations and I aim at plotting the median external standard error per star which would be 1 degree, and the name should be Ptolemy, not Hipparchus. I have changed the value to 3600” and the name to Hipparchus/Ptolemy; it would be too sad to omit Hipparchus' name entirely.

On 27 August 2008, F. Mignard informed me of an unpublished study made in 2001 where he compares Ptolemy's catalogue with Hipparcos data. He finds a standard deviation of 0.5 degree using a robust estimator. A following discussion by mail between Mignard and Arenou showed that some issues deserve a closer study. For the time being I will stick to the one degree error, according to Shevchenko (1990), published recently in a refereed journal.

Landgrave of Hesse 360” about 1570
It appears that Tycho Brahe's a little older colleague, Wilhelm IV, called The Wise, Landgrave of Hesse-Kassel (1532-92) was able to measure positions much better than Hipparchus/Ptolemy in the Antique. Eichhorn p. 101 gives an rms error of 6' for the catalogue of 1004 stars, published in 1594 by Wilhelm and Christoph Rothmann.

Tycho Brahe 60” at 1586
The accuracy of Tycho Brahe's instruments has been studied by Wesley (1978). For the best of Tycho's nine fundamental stars, he finds an accuracy of 25” for individual measurements with some of the six instruments he considered. He says: “For the majority of the stars that appear in Tycho's final catalogue the overall accuracy may be much less; for there were fewer measurements taken with them...”. I adopt 60” as still plausible for the median standard error.

Sources for the diagram after 1700 AD

The following text from 2008 is only altered with respect to Hevelius, Rømer, Flamsteed, and Lalande and the cancellation of the SIM mission. The accuracy of an observation catalogue of positions is plotted at the mean epoch, while the catalogues FK5 and PPM, compiled from observations with many instruments, are plotted at the year of publication. The Jenkins compilation of parallaxes is also plotted at the year of publication.

Flamsteed 40” at 1700
Eichhorn gives an rms. error for Flamsteed of 2”, which must be a misprint for 12” since that is what some others assume. Chapman (1983) cites Schuckburgh and Pearson (respectively 1793 and 1819) for an error of 10”-12”, here is probably where many others took the values.

Other values are quoted by Nielsen (1968). He quotes Argelander (1822) for finding an internal mean error about 7” and an external about 60”. He quotes Piazzi (1813) for a long statement which I condense to: an external mean error of 30” and individual errors exceeding 60”. This together, I settle on 20” for the catalogue which differs a lot from the conventional 12”, but I cannot avoid it.

Added in 2016: Lequeux (2014a) gives standard errors of 47.8" and 31.3" for RA and Dec respectively based on 220 randomly selected stars of which 195 were retained. We adopt 40" for the diagram instead of 20" for the 2008-diagram.

In 2016, I became aware of a paper by Blitzstein (1997). The author discusses the random and systematic (bias) errors of Flamsteed's Mural Arc and how the published observations could be corrected by proper calibration in our time. But the purpose of the present report on astrometric accuracy is to show the accuracy of catalogues available to astronomers as originally published, in this case 1725, the paper by Blitzstein therefore did not lead to any change of the numbers available in 2008.

Flamsteed is credited by Blitzstein thus: "He was the first to develop and utilize systems for meridian transit observations using the newly developed pendulum clocks and he was the first to investigate the bias errors of his measuring system. He was the first to use telescopic sights for this purpose. He developed the first methods for reducing meridian transit observations to right ascension and declination for a specific epoch. His observations, after correction for known errors, were about an order of magnitude more accurate than any previous ones."

Rømer 4"

During the same years as Flamsteed, Ole Rømer developed novel astrometric instrumentation with telescopic sight, notably the meridian circle which became the fundamental astrometric instrument for centuries. Rømer's positions were use by Tobias Mayer, see below, to derive proper motions which were used by W. Herschel to derive the solar apex.

Ole Rømer developed the transit instrument and the meridian circle, but most of his observations were lost. Rømer's only surviving observations with the meridian circle in 1706, written in the so called Triduum (three nights), were published by Horrebow (1735). The reduced positions have been calculated and were used but were never published. They are discussed by Nielsen (1968) where further references are given. On three nights, 250 transits were observed of 88 stars, the Sun, the Moon, and all the planets known at that time, from Mercury to Saturn. Nielsen has determined the errors of a subset of the star positions by comparison with newer observations and finds external errors in RA of 3.4” and in Dec. 4.5”, which I combined to the single number 4” in 2008 for a position with an average of 2.6 observations. This seems to agree with a statement by Piazzi (1813), according to Nielsen.

More recently, a paper (in Danish) by Fabricius (2011) discusses the accuracy of individual observations by Rømer of stars and planets. His Figure 3 shows that the external error of an observation of the declination of a star is about 10". It appears that much of this is due to errors of the circle divisions. If they could have been determined the error of an observation would have been about 5". The external errors of stellar transit times can be estimated from Figure 4 to about 0.4 s, i.e. 6" in RA. The observations of five planets are shown in Table 1 *after* correction for division errors and for clock errors. Fabricius concludes that the error of repeated observations of the same planet is 3-4" and that the total error is about 6".

On this basis we may conclude that an external standard error for a single observation of 6" could have been obtained with Rømer's meridian circle if clock corrections had been applied as would have been possible and if the circle had been divided as Rømer had planned, which it was not. This is in fair accordance with the external errors quoted above from Nielsen (1968) when taking into account that Nielsen analysed observations from two narrow intervals of declination where the division errors have played a minor role.

Nielsen quotes Bessel (1824) after he had read the book by Horrebow (1735) about Rømer, here translated from German: Bessel draws attention to how much could have been reached already at Rømer's time: "... if the road shown by Rømer had not been left again."

Lalande 4” at 1795

3” from Mineur-1939, 3” from Turon-2007. Arenou (2008) confirms the 3” and calculates the mean epoch to 1795. Lindhagen in 1849AN.....28..129L derives that the number of different stars in Lalande's catalogue is perhaps 40,000, much smaller than the number of entries in the catalogue of about 50,000. The accuracy of 3” can only be valid for the best part of the positions in the catalogue, which is known to contain many errors. F. Mignard notes in a recent mail: “... the Histoire Celeste is a very valuable and extensive description of the sky around 1800 (celebrated as such for example by Olbers), but of low interest in term of astrometric quality. ... In short it is the equivalent in the early 1980 of the SAOC compared to FK4 or GC. ... Histoire Celeste is an astronomical landmark for sideral astronomy, but not for astrometry.”

Added in 2016: Lequeux (2014a) gives standard errors of 4.3" and 3.8" for RA and Dec respectively based on 198 randomly selected stars of which 188 were retained. We adopt 4" for the diagram instead of 3" for the 2008-diagram.

Argelander 0.9” at 1856

Eichhorn p. 147, gives a mean error of 0.9” for Argelander's large catalogue of 33811 stars from 1867. On p. 143 Eichhorn explains that he assumes that two observations were always combined to give the published position. In the first versions of my diagram I took Argelander's catalogue of 26425 stars from 1844 for which the error is given as 1.1”. I think it is more appropriate to take the larger catalogue, but it makes no significant difference for the diagram.

FK5 62 mas plotted at 1988

The catalogue FK5 states on p. 8 an average “mean error” of individual positions at the mean epoch about 23 mas and of proper motions 0.75 mas/yr. This implies an individual standard error in 1991 of 38 mas, but the error is in fact 62 mas, or 1.6 times larger, as may be concluded from a study by Mignard & Froeschlé (2000) who have compared FK5 with Hipparcos. Their tables 3 and 4 show the local systematic differences, averaged over 230 square degrees, between Hipparcos and FK5 positions at the Hipparcos epoch of 1991.25. From the tables we find an rms value of 58 mas. Adding the 23 mas gives 62 mas which we consider to be a reasonable estimate of the individual standard error in 1991 and which is therefore adopted for the last column in Table 2.

We tentatively assume that the above factor 1.6 should be applied to the errors on p. 8 giving 40 mas instead of 23 for the error of positions at the mean epoch which is then adopted for FK5 in Table 1. The individual proper motion error becomes 1.2 mas/yr instead of 0.75 and this is adopted in Table 2.

PPM 144 mas plotted at 1992

For Table 2 the standard errors of positions and proper motions are adopted for north and south as given in the catalogue, volumes 1 and 3. This combines to 144 mas for positions for the whole catalogue. It is essential to include PPM in the diagram because it is the last large purely ground-based catalogue before the Hipparcos results appeared. It is therefore more fair to take PPM for comparison with the large catalogues based on space observations, rather than to take the FK5 containing only the very few, very best observed bright stars.

Tycho-2 60 mas at 1991

Tycho-2 includes positions and annual proper motions, derived from Tycho-2 positions and more than 140 ground-based position catalogues, but no parallaxes. The median standard error for positions of all stars in Tycho-2 is 60 mas, and for stars brighter than 9 mag it is 7 mas. The median error of proper motions is 2.5 mas/yr.

Hipparcos 1 mas at 1991

Hipparcos obtained the median accuracy of 1 mas for positions, annual proper motions and parallaxes of 120 thousand stars.

Roemer 0.1 mas at 1992

The Roemer space mission of 1992 (Høg 1993) proposed to use CCDs in TDI mode and promised a factor 10 better accuracy than Hipparcos for many more stars, viz. 0.1 mas as median accuracy for the 45 million stars brighter than 15 mag, and an error better than Hipparcos for the 400 million stars brighter than 18 mag. It is included in the diagram because the Roemer idea led to the Gaia mission, and to the studies of DIVA and FAME. The use of CCDs as modulation detectors was proposed by Høg & Lindegren (1993) but this idea was not further pursued after the superiority of CCDs in scanning mode had been realized.

Gaia 10 and 180 microarcsec at 2015, two dots plotted

Table A. Median astrometric accuracy for Gaia as function of magnitude. Courtesy of Jos de Bruijne.

(1)	(2)	(3)	(4)	(5)
G=06.0-13.0	10.200	8	6	4
G=13.0-14.0	12.700	11	8	6
G=14.0-15.0	24.567	17	13	9
G=15.0-16.0	50.340	27	20	13
G=16.0-17.0	94.486	42	32	21
G=17.0-18.0	170.625	67	51	34
G=18.0-19.0	308.589	112	84	56
G=19.0-20.0	562.010	196	147	98

Column (1) G magnitude range.

Column (2) Number of stars in the G magnitude range (unit is million stars); the sum of column (2) is 1233.517 which is the total number of stars used in the Gaia galaxy model (1.2 billion).

Column (3) Median parallax error for all stars up to the faint magnitude of the magnitude range (unit is muas).

Column (4) Median proper-motion error for all stars up to the faint magnitude of the magnitude range (unit is muas per year).

Column (5) Median positional error for all stars up to the faint magnitude of the magnitude range (unit is muas).

Example: "G=17.0-18.0", "column (4) = 51 muas per year" means that the median proper-motion standard error for all stars brighter than G=18 mag (all stars in the range G = 6-18 mag) is 51 muas per year.

The Gaia mission is scheduled for launch in 2011 and a factor of 100 over Hipparcos is predicted for the 23 million stars brighter than 14 mag, i.e. 10 microarcsec median error. The median accuracy for parallaxes and annual proper motions of the 1200 million stars in the final Gaia catalogue is expected to be about 180 microarcsec, much better than the accuracy of Hipparcos. This appears from the above Table A, including explanations by J. de Bruijne.

Added in 2016:
Gaia was in fact launched in December 2013. Up-to-date performance values of Gaia are given at: http://www.cosmos.esa.int/web/gaia/science-performance. We quote: At the time of the In-Orbit Commissioning Review (July 2014), the predicted end-of-mission parallax standard errors σ_π, averaged over the sky for a uniform distribution, for unreddened B1V, G2V, and M6V stars are:

	B1V	G2V	M6V
V-I$_C$ [mag]	-0.22	0.75	3.85
Bright stars	5-16 µas (3 mag < V < 12 mag)	5-16 µas (3 mag < V < 12 mag)	5-16 µas (5 mag < V < 14 mag)
V = 15 mag	26 µas	24 µas	9 µas
V = 20 mag	600 µas	540 µas	130 µas

These numbers could be updated after September 2016 when Gaia-DR1 was released.

The first release, Gaia-DR1, of astrometric data from Gaia came in September 2016 containing the mean stellar positions and magnitudes from the 14 months of observations, and proper motions from the combination of Gaia data with Hipparcos prior information (HTPM), see Michalik et al. (2015). Simulations of TGAS, the Tycho-Gaia astrometric solution, suggest that the accuracy of the resulting astrometry for the Tycho stars will be similar to the Hipparcos Catalogue, and possibly significantly better depending on the exact scenario of the number of Gaia observations available, dead time intervals, calibration, etc. Moreover, the dataset would be almost complete to about V = 11.5, or 3–4 mag fainter than the survey part of the Hipparcos Catalogue.

SIM 3 microarcsec - Information follos as in 2008,
but the SIM mission project was cancelled by NASA in 2010 - therefore SIM is omitted in the 2016 diagram

The dot for SIM has been placed at 3 muas with 10,000 stars, although 1300 would be more correct at this accuracy, but space in the diagram is limited. In fact, a dot at 10 muas with 10,000 stars and another dot at 3 muas with 1300 stars would be more correct.

The NASA interferometric mission (Unwin et al. 2008, Shao 2008) is expected to give global astrometry with few microarcsec accuracy after a five year mission down to 20 mag for more than 10,000 stars. Table 7 in Unwin et al. (2008) gives expected performances, especially 4-20 muas for 10,000 stars of -1.4-20 mag in key projects and 3 muas for 1300 stars of 9-10.5 mag in the astrometric grid.

Narrow angle accuracy of 1 microarcsec per 20 minutes integration is predicted for stars of 6-9 mag. The SIM project has passed all milestones in over ten years of design and development, but is not yet an approved mission and the launch will be after 2014-15.

Bradley-aberration 1” at 1728
The points marked “parallaxes” might be labelled “small-angle astrometry” or “relative astrometry”, and all ground-based measurements of parallaxes are of that kind. This is about ten times more accurate than large-angle astrometry required for the stellar positions in the diagram. The first such point is “Bradley – aberration” shown at 1.0 arcsec, the accuracy which Bradley obtained for the constant of aberration with his zenith sector. According to Arenou (2008) using Flamsteed observations (1689-1697) the precision of

aberration can be found within 1.1". This information is from F.G.W. Struve, *Ueber Doppelsterne nach den auf der Dorpater Sternwarte mit Fraunhoffers grossem Fernrohre von 1824 bis 1837*, 1837, page 95: http://books.google.com/books?id=MEMJAAAAIAAJ&pg=RA2-PA95&lpg=RA2-PA95&dq=flamsteed+aberration+1689+1697&source=web&ots=0YS4rHY2eg&sig=1Kh53ZgrXvLb1BGUeLjbLXbYhhc&hl=fr&sa=X&oi=book_result&resnum=1&ct=result

Bessel 60 mas at 1838
The accuracy of ground-based parallaxes begins with Bessel's single star in 1838. The 60 mas is based on the analysis below for Table 3. Previous diagrams had, e.g., 60 mas in Høg-1995 and 300 mas in Mineur-1939.

Jenkins 15 mas plotted at 1952
This accuracy for the parallaxes in Jenkins' catalogue was derived by Hertzsprung (1952).

USNO 0.6 mas at 2008
At the U.S. Naval Observatory in Flagstaff, relative parallaxes for 357 faint stars has been obtained with a standard error of 0.6 mas, according to W. van Altena/ C. Dahn (2008 priv. comm.).

The above sources are usually NOT REPEATED below for the Tables 1, 2 and 3 in Høg (2017b) "Selected Astrometric Catalogues".

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6 Sources for Table 1 of positions in Høg (2017b)

Hevelius 2'
Eichhorn (1974) does not give a value for the accuracy of Hevelius. Chapman (1983) p. 136, gives the values 15" to 20" with a reference to Schuckburgh and Pearson from respectively 1793 and 1819 which I have not read. But I adopte the value 20" for my Table 1 in 2008. Chapman in fact plots a value at 25". - In 2016 the value was changed to 2', see about Hevelius above.

Lacaille 6"/30"
6" from Mineur-1939; unfortunately I know no primary source. See more below under Piazzi.

Added in 2016: Lequeux (2014a) gives in his Table 1 details about astrometry by La Caille, the standard deviation for fundamental stars and zodiacal stars is about 6" and it is 30" for the survey in 1752 of 9766 stars of the southern sky.

Added in 2017: According to van Gent, priv. comm., the 'complete' version of this catalogue with 9766 stars was not published in 1763 – in that year a catalogue with only 1942 stars was published – but in 1847 (i.e. much later) by the British Association for the Advancement of Science. Lacaille's 1763 catalogue is here http://www.e-rara.ch/zut/content/titleinfo/152572 The 1847 catalogue is here https://archive.org/details/catalogueof9766s00lacarich

Added in 2017:
Tobias Mayer 6"
A study by Wittmann (2012) gives new information and I quote the abstract - this paper in German is not available in ADS. **Abstract by Wittmann**: "In 1756-1757 Tobias Mayer observed the topocentric

coordinates (time of :transit, zenith distance) of approximately 1000 zodiacal stars (i.e. stars near the ecliptic) using Bird's 6-ft mural quadrant at Göttingen Observatory. By applying corrections for refraction, precession, aberration, and misalignments of telescope and quadrant, Mayer derived the mean heliocentric positions (right ascension, declination) of 998 stars. He presented his hand-written catalogue to the Göttingen Society of Sciences (the later Academy) in 1759. Mayer's catalogue of zodiacal stars was first published in printed form by Georg Christoph Lichtenberg in 1775, thirteen years after Mayer's death. Using modern data, including satellite data, for each individual star, Mayer's positions of 998 stars (one of which is actually is the planet Uranus) have been calculated and compared with the observations; their average accuracy is approximately ±6" (for details see Section 4)."

Tobias Mayer used his own positions and those of Ole Rømer to compute proper motions of 80 stars in 1760, the first mass production of proper motions and based only on modern observations. Edmund Halley had in 1718 discovered the proper motions of three stars using antique and modern positions. William Herschel used these motions to compute the solar apex motion in 1783.

Wittmann (2012) does not discuss the accuracy of Mayer's proper motions, but he gives the values of proper motions for Sirius and Arcturus and it appears that they deviate by about 0.3"/yr from modern values.

Bradley/Auwers 1.1”
Turon-2007 shows 2” for Bradley/Bessel. This is in accordance with the following analysis.

Rather than Bessel's version the one by Auwers should be used, thus Bradley/Bessel/Auwers, which has probably been used for the German fundamental catalogues from Auwers' FC to FK5. Bradley's precision was in general 1", if one should believe
http://www.flamsteed.info/fasbradley_files/page0002.htm.

Eichhorn's table II-1 on p.66 gives internal errors of a *single observation,* which is not stated by Eichhorn, but it is by Cohn (1907b) on p.269. The errors are 0.16 s and 1.92” for Greenwich in 1755, i.e. Bradley. But table II-3 gives 0.16 s and 1.3” for one observation by Bradley. Using the formulae in the footnote to table II-1 give however 0.18 s = 2.7” for Dec=0 and 1.92” for zenith distance =0. Rounded to 2” for Bradley/Bessel in accordance with Turon2007. The value is for a single Bradley observation, which may apply to the bulk of the 3222 stars in the catalogue. He did probably make many more observations per star for those few hundred used in the German fundamental catalogues.

It is not clear from Cohn (1907b) or Eichhorn whether this accuracy refers to Auwers' reduction of Bradley/Bessel, and this makes a difference. The version Bradley/Bessel/Auwers obtains an increase of weights compared with Bradley/Bessel of the factors 1.75 in RA and 1.4 in Dec, according to Auwers as quoted by Cohn (1907b), p.269. This would lead to 2”/sqrt(1.6)=1.6”. This is an example how difficult it can be to get a half-way reliable standard error for a catalogue position in Table 1.

Very recently, however, I received Brosche & Schwan (2007) from the first author. It contains a direct comparison of Bradley/Auwers and Hipparcos. For 2450 catalogue values out of the 3268 entries the rms values are 1.2” and 1.0” for respectively RA and Dec. This gives 1.1” for my Table 1, in reasonable agreement with the above 1.6”. The weight has then been calculated using for simplicity the N=3222 in the preceding column, although a smaller number would be more correct since only N=2450 were good enough for the comparison.

Added in 2016: Lequeux (2014a) finds a standard deviation of 2.3" for Bradley/Bessel, but I will keep the 1.1" for Bradley/Auwers.

Piazzi 2.5”

1.5” from Mineur-1939; unfortunately I know no primary source. F. Mignard wrote in a mail: “The most interesting report I found [on Lacaille and Piazzi] is by R. Grant (History of physical astronomy (London 1852) in chap. XIX on the Catalogues of fixed stars from Hipparchus to his time. He praised very much Lacaille care in obtaining absolute measurements on few reference stars. Same opinion about Piazzi work in Palermo using again the 36 fundamental stars of Maskelyne before and building himself a fundamental catalogue of 120 stars before forming his catalogue of 7600 stars. Every stars has been observed several times and "this work is justly considered to be one of the most important that has ever been executed by a single individual".”

Added in 2016: Lequeux (2014a) finds a standard deviation of 2.5" which replaces the 1.5" given in 2008.

Küstner 1908, AC, Stoy 1968, SAOC 1965

Standard errors are taken from Eichhorn p.157, p.279, p.162, p.209.

USNO 1920 and USNO 1952, about 0.15 internal errors

Standard errors are taken from the references in Høg (2017b). Only internal errors are given in the publications as derived from the repeated observations of the same star on different night. These internal errors are divided by sqrt(n) for inclusion in Table 1, because no external error is available. The details for these catalogues are as follows.

USNO (1920) gives the typical internal errors of one observation for RA and Dec on p. A79 and A139 as 0.50” and 0.48”, respectively, which combine to 0.49”. The probable errors used by USNO in those year are converted to standard errors by multiplication with 1.50. With n=10 the 0.15” in Table 1 is obtained.

USNO (1952) gives the typical internal errors of one observation for RA and Dec on p. 375 and 377 as 0.32” and 0.45”, respectively, which combine to 0.37” (as average of the weight from each coordinate). With n=6 the 0.15” in Table 1 is obtained.

These two catalogues are based on respectively 45,000 and 31,000 meridian observations, both obtained in eight years in Washington DC around 1907 and 1945. The development in this period improved the internal error of an RA observation from 0.50” to 0.32” while an observation of Dec stayed about 0.46”.

GC 0.15” and 10 mas/yr

According to Eichhorn (1974) p. 204: “... in the General Catalogue the accidental rms. errors of the positions vary strongly from one star to the next. However, at the epoch they are on the average about 0.15” in both coordinates, and rise to an average of at least 0.70” in 1965 because of the uncertainties of the proper motions (Schlesinger and Barney 1939a).”

Since the (mean) epoch for GC is 1900 this implies a standard error of the proper motions in GC of sqrt(0.7^2-0.15^2)/65 = 0.0105”/yr. The value of 10 mas/yr is adopted for Table 2, but is not stated by Eichhorn; it is however in accordance with the error given by Scott (1963). For Table 1 the value 0.15” is adopted.

Perth70 0.15” external error
Standard errors are taken from the reference in Høg (2017b). Internal standard errors of one observation reduced to zenith is 0.17” and 0.27” for RA and Dec, respectively, cf. Eq. 15, and 0.10 mag for the photoelectric photometry in the visual band. External errors have been derived from observations of circumpolar stars, taking asymptotic errors into account. The typical standard errors of a catalogue position for a program star with four observations are accordingly 0.12” and 0.20” in respectively RA and Dec. This combines to an error per coordinate of 0.15”, adopted for the Table 1.

These internal errors of one Perth70 observation obtained about 1970 are about half the size of those in USNO (1952) and 100,000 such observations were obtained in 5 years in Perth, Western Australia, compared with the 31,000 in 8 years in Washington DC. Thus, a considerable progress in meridian observations were achieved in those years using the photoelectric semi-automatic instrument of the Hamburg-Perth Expedition.

The error of a catalogue coordinate is given as 0.15” in both cases, but they cannot be compared directly because the USNO error is an internal error, the Perth70 error is external.

CMC1-11 in 1999 and CMC14 in 2005
Information from the web supplemented by correspondence with D. Evans is shown in Table 1 and explained in Høg (2017b). The CMC1-11 catalogues were obtained with a photoelectric slit micrometer, similar to the one used for Perth70, but with automatic control of micrometer and telescope giving a much higher efficiency. Observed in the better seeing on La Palma and during 14 years instead of 5 years for Perth70 the weight of the catalogue is larger by a factor 30. This is the last meridian circle catalogue in the table where large-angle astrometry is performed. The CMC14 is observed with CCDs in drift-scan mode and the reference stars of the Tycho-2 Catalogue are used for the resulting small-angle astrometry.

USNO-B1.0 2002, UCAC2 2003, GSCII 2005
Information from the web supplemented by correspondence with S. Urban.

2MASS
The 2MASS all-sky catalogue was obtained by two highly automatic telescopes with 1.3 m aperture equipped with HgCdTe detectors sensitive in the J,H,K bands (1-2 microns) with a limit of 17 mag in J. An accuracy of 0.5” for positions was expected, in fact 0.08” was achieved according to N. Zacharias.

7 Sources for Table 2 of proper motions

Auwers' FC and NFK
For lack of better knowledge, the values are estimated, based on FK3 and N30, therefore the question mark after each of the values.

FK3, GC and N30
Scott (1963) gives an overview, including the proper motion errors for FK3, GC, and N30.

FK4 and FK5
The individual proper motion error becomes 1.2 mas/yr for FK5 instead of 0.75, as derived above under FK5. The error given for FK4 is simply set a bit larger, 2 mas/yr, for lack of better knowledge.

SPM3, UCAC2, USNO-B
All data were received from N. Zacharias in October 2008.

More on proper motions from Arenou
Arenou (2008) mentions two important catalogues: "One led to the discovery of the astrometric binaries: I think that Bessel had 38 stars among which 36 zodiacal stars from Bradley as first epoch (1755) or Maskelyne?. Then, I understand that Argelander had proper motions for 560 stars in 1835 (see 1837MNRAS...4...82A) of which he used 390 to confirm the solar motion."

More on proper motions from Zacharias
"Traditionally proper motions of stars have been determined by comparing absolute positions (on a fundamental system) at different epochs. With the improvement of the photographic technique in the middle of the 20th century it became possible to image distant galaxies in a sufficient number to determine absolute proper motions field by field with differential, small angle measures of pairs of plates taken many years apart, covering large areas of the sky for galactic dynamics studies (Wright 1950). This lead to the Northern Proper Motion (NPM) program using the Lick 50 cm double-astrograph (Klemola et al. 1987) and its southern counterpart, the SPM, using the Yale / San Juan instrument of similar design (Girard et al. 1998). These plates, spanning an epoch difference of about 25 years were initially measured with slow but accurate PDS machines for selected stars. By the turn of the century all applicable plates were measured with the PMM at the Naval Observatory Flagstaff station to obtain positions of all stars to 18th magnitude. Reductions are still in progress as part of the UCAC3 effort. Even after no photographic emulsions are any longer in use in astrometry, the development of plate measure machines progressed in the late 20th and early 21st century to allow extraction of all astrometric (and photometric) information available in those data materials."

On reference catalogues
The fundamental catalogues, Auwers FC to FK5, contained too bright and too few stars, FK5 only 1535, to serve directly as a reference catalogue for the reduction of photographic plates. Special observing campaigns were therefore organized to provide denser nets of reference stars for the various photographic surveys, e.g., the AGK3R of 21,499 stars was observed with meridian circles in the 1950s while the AGK3 survey of the northern sky was made. Subsequently, a list of 20,495 Southern Reference Stars was defined and these stars were observed in an international collaboration agreed at the IAU Assembly in Moskau 1958. The resulting SRS catalogue combined with the AGK3R was called International Reference Stars (IRS) which was completed in the 1990s.

The more detailed history of the IRS and the larger ACRS, Astrographic Catalog Reference Stars, is told in the recent message from T. Corbin which I have slightly edited.

"The IRS project originated in the 1960's when T. Corbin was asked to derive proper motions for the observed positions being compiled from the AGK3R observing program. This was to allow the AGK3R positions to be brought to the epochs of the individual AGK3 plates. Only meridian circle catalogs were to be used in order to avoid the color and magnitude terms that older astrograph catalogs would introduce. Catalogs that had been observed using screens were employed to extend the FK4 system to fainter magnitudes, and that extension provided the reductions for the other catalogs. The same thing was done for the SRS.

The IRS then resulted from combining the AGK3R and SRS, each reduced to FK5, and, using the same approach for reducing the older catalogs, computing new mean positions and proper motions on the FK5 system. The FK5 Part II was compiled by combining the FK5 based positions and motions for both FK4 Sup stars selected at Heidelberg and IRS selected for the list at USNO.

ACRS grew from a USNO collaboration with P. Herget in the early 1970's to get improved plate constants for the Astrographic Catalog. The Bordeaux zone was selected, and Corbin compiled a more dense catalog for this part of the sky by combining IRS data with astrograph programs. Herget obtained a significant improvement in the plate solutions, and this showed that compiling such a catalog on a global scale for the reductions of all AC zones would be worth the effort.

The ACRS (Astrographic Catalog Reference Stars) is basically an extension of the IRS. Particular attention was given to minimizing the systematics in order that the 320,211 stars would represent the FK5 system at the CdC epochs. The PPM was being compiled at Heidelberg at about the same time. PPM includes the AC data, and this is the main difference between it and the ACRS. Both catalogs are based on IRS.

S. Urban used the ACRS database, in combination with Tycho-1 to create a new version of ACRS that then gave an improved set of results for the AC zones. This was all combined to produce the ACT catalog which was quickly superseded by a new version of the proper motions using Tycho-2 results. These were combined with the Tycho-2 observed positions to give the final Tycho-2 Catalogue.

IRS contains 36,027 stars, 124 catalogs were used
errors of proper motions - 4.3 mas/yr in RA and 4.4 mas/yr in DEC
errors of positions - 0.22 arcsec in both coordinates

ACRS contains 320,211 stars, 170 catalogs were used
errors of proper motions - 4.7 mas/yr in RA and 4.6 mas/yr in DEC
errors of positions - 0.23 arcsec in both coordinates at 2000
“ end of quotation from Corbin in 2008.

8 Sources for Table 3 of trigonometric parallaxes

The three first parallaxes
This is here at first retold after Stephen Webb (1999) p.71, and then after F.W. Bessel (1838 and 1840), in both cases abbreviated, followed by my conclusions about the standard errors of the three values as adopted for Table 3.

Quoting Webb (1999): The parallaxes were:
Bessel 0.31” for 61 Cygni (modern value from Hipparcos: 0.287”)
Henderson 1.26” for alfa Cen (Hipparcos : 0.742”)
Struve 0.2619” for Vega (Hipparcos: 0.129”).
(Webb gives the same modern values for the first two stars, but 0.125 for Vega!)

Struve studied Vega with a wire micrometer on the big refractor in Dorpat. Struve made 17 observations during 1836 which gave a parallax of 0.125” with an uncertainty of 0.05”. This was published in 1837. He promised to make more observations and published in 1840 the results of 96 observations made up to 1838. The parallax he obtained this time was 0.2619, more than twice the original result, which cast doubt on both values.

Bessel, meanwhile, studied 61 Cygni with a Fraunhofer Heliometer in Königsberg, using two nearby companions. He began observations in September 1834, but this was interrupted by other work. He returned to the task in 1837 and made 16 or more observations every clear night. As result of his analysis at the end of 1838 he announced a parallax of 0.31” with an error of 0.02”.

Henderson studied alfa Cen with a mural circle from Cape. He completed his observations in 1833, and analysed them upon his return to Scotland later that year. He arrived at a parallax of 1.16” with an error 0.11”. Before publishing his results, however, he asked a colleague to check his work. In the end he published several weeks after Bessel.

More now from Bessel : Bessel (1838) explains his observations and reductions and gives first the annual parallax derived from the star *a* at 8' distance and from star *b* at 12'. They are respectively 0.3690” +-0.0283” and 0.2605” +-0.0278”. The combined solution from *a* and *b* gives 0.3136” +-0.0202”.

Knowing today the very accurate modern values for all three stars, considering them to be the true values, we can derive the true residuals. For Bessel (1838) it is O-C= +0.026”, in good accordance with Bessel's mean error of 0.0202”. That would have led to 20 mas for Table 3, but recently I learnt (Arenou 2008) that two years later, Bessel (1840) gives the value 0.3483” with the mean error 0.0141”, at 0.061” or more that 4 sigma from the true value. I therefore finally adopt 60 mas for the Table 3, also because Bessel's final value will have been the most trusted at his time. In previous diagrams are found 60 mas in Høg-1995 and 300 mas in Mineur-1939.

Struve and Henderson: For Struve's final value O-C= 0.2619”-0.129”=0.133”. This is our best estimate of his standard error, and this estimate has a relative standard error of 1/sqrt(2f)=0.71 since there is f=1 degree of freedom. I adopt 100 mas for Table 3.

Henderson's value gives O-C=1.26”-0.742”=0.518”, much larger than his own claimed error of 0.11”. I adopt the error of 500 mas for Table 3.

Review and catalogue by Oudemans in 1889

I quote Mignard from a mail in Aug. 2008: “I came across the attached reference of interest for your current investigations. This compilation of parallaxes was mentioned in the 'Traite d'Astronomie Stellaire' of Ch. André published in 1899. This is given by him as the Catalogue of the known stellar parallaxes. An interesting point is that in 1899, the analysis of a large number (55) of determinations for 61 Cyg led to pi= 0"44.” - end of quote from Mignard. This catalogue by Oudemans, see Mignard (2008b), is dated 1889, and the “best” value for 61 Cyg was 0.40”, if I read from Tabelle II, i.e. 0.11” too large.

Bigourdan 50/30 mas and Russell 40 mas

The values for both are placed in brackets because they are internal, formal errors. A catalogue by Bigourdan (1909) lists trigonometric parallaxes for about 300 stars, a few with up to 40 observations. The consistency of multiple observations indicates a precision about 50 mas per observation, and a median precision of 30 mas may be inferred for the about 200 stars having more than one observation. Many observations are shown (by bold face) to be the average of several measurements by the same observer, including most of the 100 with only one observation.

The catalogue by Bigourdan is very complete for its time, and may be of interest for further analysis. It is made available in a file, collected by Mignard (2008a).

Russell (1910) presents 52 new photographic parallaxes and claims a standard error about 40 mas.

Schlesinger 15 mas
This is my estimate, based on the value for Jenkins.

Jenkins 15 mas
The 15 mas are from Hertzsprung (1952). It is perhaps interesting to note that this catalogue from 1952 containing photographic parallaxes of 5800 stars has nearly the same accuracy as claimed in 1840 (see above) by Bessel for 61 Cygni, with heliometer. But of course, Bessel observed only one star, with utmost care and with an excellent instrument, and later observations with heliometers gave a much larger parallax. To reach 15 mas and much smaller systematic errors for thousands of stars required an enormous effort in development and implementation. Strand (1963) gives an overview of parallaxes at that time.

Van Altena 10 mas
Bill van Altena has seen the whole Table 3, made no remarks to the rest of it either, and has thus agreed to the information about modern photographic parallaxes.

Hipparcos 1 mas
Hipparcos obtained a median standard error of 1.0 mas for parallaxes.

USNO 0.6 mas and Hubble 0.24 mas
A better accuracy than 1 mas has been achieved from the ground and with the Hubble Space Telescope for several hundred much fainter stars. This informations was received in correspondence with W. van Altena and the informers are named in the table.

Parallaxes according to Westfall (2001)
The numbers of well-measured stars by (year) are about: (1839) 3, (1850) 6, (1862) 10, (1888) 25, (1901) 38. The same source mentions a 1912 catalogue with the parallaxes of 244 stars, determined as follows: 8 with filar micrometers, 83 with meridian transits, 39 by photography, 3 by spectroscopy, and 111 with heliometers.

More on parallaxes from Arenou (2008)
“About the number of parallaxes and the reference by Westfall (2001), one can find that in 1846, Peters has 8 parallaxes (Polaris, Capella, i Ursae maj, Groombridge 1830, Arcturus, Véga, alfa Cygni, 61 Cygni), observations between 1842 and 1843, cf FGW Struve, "Études d'astronomie stellaire", 1847, p 94 (vs Westfall: 1850: 6). In 1889, Oudemans, 1889AN....122..193O, there are 46 stars (vs Westfall: 1888: 25). And then, "The Parallaxes of 3650 Stars of different galactic latitudes, derived from photographic plates", 1908PGro...20....1D, Donner et al.”

Present-day catalogues for astrometric data
A list of presently widely used or well known catalogues for astronomical and especially astrometric data is provided by Zacharias et al. (2004). The list is intended to give users some basic information with regards to the content and usefulness of each. Within each section the catalogues are listed with progressively more and fainter stars but generally with decreasing accuracy.

9 Appendix: Other diagrams of astrometric accuracy

Here follow a series of diagrams, placed in the sequence I have first seen them. This is the sequence in which the reader can most easily follow the development of the diagram ending with the above Fig. 1. But it is not the sequence in which the diagrams have been published. The first two of the kind were published in 1939 and 1983, but they only came to my knowledge in respectively March and May 2008. Only then did I understand what had made the confusion; these two diagrams by respectively H. Mineur and A. Chapman are shown as Figures 9 and 10.

The diagram Hipparcos-1985 puzzled me in 1985 because a nearly linear development is indicated over 450 years from Copernicus to Hipparcos, even the last piece of 150 years from Simms to Hipparcos fits this line! This cannot be correct, but we had other more urgent tasks in 1985 than to dig deeper here. Four years later the same diagram was used, Hipparcos-1989. In general in these diagrams, one should never draw lines from one point to the next since this indicates that one could interpolate. But it is appropriate to draw a longer line in order to indicate a trend, as has been done in later diagrams, e.g. Høg-1995 and ESA-1998.

Then I saw the diagram Kovalevsky-1990 presented a year later, very different, but again I was puzzled. I wanted to dig deeper, but five years passed before I found the time to make Høg-1995 which was immediately accepted in the Hipparcos Science Team. The jumps in accuracy at Tycho Brahe and at our Hipparcos satellite are clearly seen. Two more versions are shown here as ESA-1998 and Høg-1995/2005.

At the symposium in Shanghai in 2007 Catherine Turon showed the diagram Turon-2007. The smooth curve could give the, I think erroneous, impression that the development had no jumps, but was completely gradual over 550 years from Ulugh Beg to Gaia, though starting to become steeper about 1950.

The diagrams are shown in the sequence they came to my eyes, in the appendix of eight pages. This appendix is placed at the following link and it is appended in the version Høg (2017d) of the present file:

www.astro.ku.dk/~erik/AccuracyAppendix.pdf

Mignard (mail of August 2008) gives references to further diagrams: “In the book of Walter and Sover (Astrometry of Fundamental Catalogues, Springer, 2000) there is one more diagram of accuracy vs. time on p. 5. The reference is given to: Schmeidler F., 1980, Die Geschichte des FundamentalKataloge, in Astrometrie und Dynamische Astronomie, W. Fricke, Th. Schmidt-Kaler, W. Seggewiss (eds), Mitteilungen der Astron. Gesell. 48, 11-23.”

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10 References

All links to my dropbox had become wrong in March 2017 but are now replaced by links to my website.

New references since 2008 are red. Some references are only mentioned in the text, and some are given only in the report Høg (2017b) "Selected Astrometric Catalogues". Here follow other references in the conventional arrangement. Some references, here and in the text, are given merely as the search code to be used at ADS: http://esoads.eso.org/ , e.g. 1840MNRAS...5...55B for Bessel (1840).

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