Statistical model of galaxies

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Abstract

The average properties and frequencies of E and S galaxies are summarized for use in the planning of GAIA. Our statistical model gives the number of E and S galaxies brighter than a given total I magnitude. The half-light radius of a galaxy is given as function of the total I magnitude. The surface brightness as function of the radius is finally provided and appears to have an accuracy of about 30%. It appears for instance that the typical surface brightness of a normal E or S galaxy at the half-light radius is $\mu_I = 21.4$ mag arcsec^{-2} and $\mu_I = 21.0$ mag arcsec^{-2}, respectively, for which significant broad-band photometry could be obtained. A galaxy of total magnitude $I \simeq 16.5$ mag can be detected with a $SNR = 10$ in the astro sky mapper (ASM1) in Astro-2. There are about 1.5 million galaxies brighter than this limit on the whole sky, mainly contained within $d = 275$ Mpc or $z \simeq 0.05$, and it is concluded that multicolour broad-band photometry can be obtained for the central parts of most of these.

1 Introduction and conclusions

Properties of galaxies are summarized. Average properties are considered, regardless of the many different morphologies of galaxies, so the results will be valid for normal E and S galaxies only. Two-dimensional surface brightness distributions are represented by elliptical isophotes whose surface brightness is a simple smooth function of radius.

Section 2 gives the number of galaxies on the sky. Section 3 gives the half-light radius. Section 4 gives the surface brightness. Section 5 compares the predictions of our model with observed brightness profiles. Section 6 discusses the possibility of detecting and photometrically measure galaxies with GAIA. Tables 4 and 5 should be useful in discussions of number and size of galaxies and fraction of sky affected by light from galaxies. Table 9 gives some typical values of the surface brightness.

The formulae and tables are believed to be valid for applications in the GAIA project where the statistical disturbance from galaxies on other objects is the issue. The formulae for surface brightness as function of the radius may also be used to estimate how faint galaxies could be detected in the astrometric sky mapper and be measured in the broad-band photometer.

2 Number of galaxies

Lattanzi (SAG_ML_3) provided some differential galaxy counts in the I band. If we approximate the diagram by a broken line in $\log N$ vs I, the counts can be written as follows

$$N = k_N 10^{c_N(I-18)} \quad [\text{number deg}^{-2}\ \text{mag}^{-1}] \ , \tag{1}$$

where values of k_N and c_N are listed in Table 1.

Table 1: Values of k_N and c_N.

I	k_N	c_N
<18	447	0.557
18-21	447	0.400
21-24	1 122	0.267

Using this approximation, the indefinite integral of N is

$$\int N\,dI = \int k_N 10^{c_N(I-18)}\,dI = \frac{k_N}{c_N \ln 10} 10^{c_N(I-18)} \; + \; constant \; . \tag{2}$$

Now we can calculate the cumulative galaxy counts in the I band, that is the number N_c of galaxies per square degree brighter than a given magnitude I_c in I. Actually, N_c is given by the following definite integral

$$N_c = \int_{-\infty}^{I_c} N\,dI \; . \tag{3}$$

The results are given in Table 4 and are consistent within 30% with those obtained in the r_f band by Metcalfe et al. (1995), when a correction $r_f = I + 0.5$ for the different photometric band is introduced.

The cumulative counts may be considered the sum of four terms, due to ellipticals, lenticulars, spirals and irregulars, so that $N_c = N_e + N_l + N_s + N_i$. For our purposes, i. e. for a study of the characteristic brightness profiles of different classes of galaxies, the lenticulars may be considered as spirals (see Section 4). In the following we need to make no assumption on the ratios between the four terms. Nevertheless we observe that both the APM Bright Galaxy Catalogue (Loveday 1996), complete to $b_j = 16.44$, and the morphological analysis of a subsample of the Third Reference Catalogue of Bright Galaxies (de Vaucouleurs et al. 1991) whose results are contained in Table 5 by Buta et al. (1994) report that the relative frequency is $N_e : N_l + N_s \simeq 1 : 5$.

3 Size of galaxies

Surface photometry of galaxies can be analysed (see e.g. Boroson 1981) by fitting an ellipse with semi-axes a and b to an outer isophote and by subsequently fitting ellipses of the same ellipticity to the other isophotes. The surface brightness profile is then obtained by plotting the isophote's surface brightness against its elliptically averaged radius $r = \sqrt{ab}$. The size of a galaxy may thus be given by its half-light radius $r_{hl} = \sqrt{a_{hl} b_{hl}}$, that is the elliptically averaged radius of the isophote that encloses half of the total light emitted by the galaxy.

Griffiths et al. (1994) plot the major axis half-light radius as a function of the total I magnitude in Figure 6. If we approximate this plot by a broken line in $\log r_{hl}$ vs I, we obtain

$$a_{hl} = k_a 10^{c_a(I-18)} \quad [\text{arcsec}] \; , \tag{4}$$

where values of k_a and c_a are listed in Table 2. Still the scatter in the plot corresponds to about 50% for galaxies brighter than $I = 20$ mag.
The half-light radius can be written as

$$r_{hl} = \sqrt{\frac{b_{hl}}{a_{hl}}}\, a_{hl} \; , \tag{5}$$

In our case (see Griffiths 1994, Table 3), the mean value of the axis ratio is $\left(\frac{b_{hl}}{a_{hl}}\right)_{\text{mean}} = 0.507$, so that

$$r_{hl} = k_r 10^{c_r(I-18)} \quad [\text{arcsec}] \; , \tag{6}$$

Table 2: Values of k_a and c_a.

I	k_a	c_a
<18	1.41	-0.198
18-21	1.41	-0.180
21-24	0.87	-0.110

where

$$k_r = \sqrt{\left(\frac{b_{hl}}{a_{hl}}\right)_{\text{mean}}} k_a = 0.712\, k_a \ , \qquad c_r = c_a \ . \tag{7}$$

Values of k_r and c_r are listed in Table 3.

Table 3: Values of k_r and c_r.

I	k_r	c_r
<18	1.00	-0.198
18-21	1.00	-0.180
21-24	0.62	-0.110

The fraction of sky inside the half-light radius of all galaxies brighter than a given magnitude in I is then given by

$$R_{hl} = \int_{-\infty}^{I_c} \pi r_{hl}^2 N\, dI \quad \text{[fraction of sky]} \ , \tag{8}$$

where

$$\begin{aligned} \int \pi r_{hl}^2 N\, dI &= \int \pi k_r^2 k_N 10^{(2c_r+c_N)(I-18)}\, dI \\ &= \frac{\pi k_r^2 k_N}{(2c_r + c_N)\ln 10} 10^{(2c_r+c_N)(I-18)} + \textit{constant} \ . \end{aligned} \tag{9}$$

Values of R_{hl} are shown in Table 4.

Table 4: Number of galaxies and their size. Differential and cumulative numbers, half-light radius and fraction of sky inside the half-light radius.

I	N	N_c	r_{hl}	R_{hl}
mag	deg^{-2} mag^{-1}	deg^{-2}	arcsec	fraction of sky
14	2.6	2.1	6.19	0.000066
15	9.5	7.4	3.93	0.000096
16	34	27	2.49	0.00014
17	124	97	1.58	0.00020
18	447	350	1.00	0.00029
19	1 120	1 100	0.66	0.00041
20	2 820	2 900	0.44	0.00053
21	7 090	7 500	0.29	0.00067
22	13 100	17 300	0.23	0.00082
23	24 300	35 400	0.17	0.00099
24	44 900	68 800	0.14	0.0012

In consideration of the possibility for GAIA to detect and measure the surface brightness profile of external galaxies, the fraction of sky R_d from which we must send data to the ground is of interest too. Let r_d be the angular radius of this "interesting" area around the centre of a galaxy; we can write r_d as the sum of two terms, that is

$$r_d = r_{hl} + \Delta r \ . \tag{10}$$

Thus R_d becomes

$$\begin{aligned} R_d &= \int_{-\infty}^{I_c} \pi r_d^2 N \, dI = \int_{-\infty}^{I_c} \pi r_{hl}^2 N \, dI + \int_{-\infty}^{I_c} 2\pi \Delta r \, r_{hl} N \, dI + \int_{-\infty}^{I_c} \pi (\Delta r)^2 N \, dI \\ &= R_{hl} + \pi (\Delta r)^2 N_c + D \ , \end{aligned} \tag{11}$$

where

$$D = 2\pi \Delta r \int_{-\infty}^{I_c} r_{hl} N \, dI \ , \tag{12}$$

and

$$\begin{aligned} \int r_{hl} N \, dI &= \int k_r k_N 10^{(c_r + c_N)(I-18)} \, dI \\ &= \frac{k_r k_N}{(c_r + c_N) \ln 10} 10^{(c_r + c_N)(I-18)} \ + \ constant \ . \end{aligned} \tag{13}$$

Values of R_d are listed in Table 5.

Table 5: Fraction of sky R_d inside a radius $r_d = r_{hl} + \Delta r$ from the centre of a galaxy.

I mag	Δr [arcsec] 1	2	3	4
14	0.000078	0.000087	0.000099	0.00011
15	0.00012	0.00015	0.00018	0.00021
16	0.00020	0.00027	0.00035	0.00045
17	0.00034	0.00052	0.00076	0.0010
18	0.00064	0.0012	0.0018	0.0027
19	0.0012	0.0026	0.0044	0.0069
20	0.0020	0.0044	0.0077	0.016

4 Surface brightness of galaxies

The surface brightness of galaxies as function of the angular distance from the centre is required for subsequent estimation. The relevant formulae are given separately for elliptical and spiral galaxies.

The total light flux of a galaxy is given by

$$F_I = 10^{-0.4I} \tag{14}$$

with an arbitrary magnitude zero-point.

The average surface brightness inside the half-light radius is then

$$\begin{aligned} \mu_{I,av} &= -2.5 \log \frac{F_I/2}{\pi r_{hl}^2} = -2.5 \log \left[\frac{10^{-0.4I}}{2\pi k_r^2 10^{2c_r(I-18)}} \right] \\ &= 2.5 \log(2\pi k_r^2) - 90 c_r + (5c_r + 1) I \\ &= a_{av} + b_{av} I \quad [\text{mag arcsec}^{-2}] \ . \end{aligned} \tag{15}$$

Table 6: Values of a_{av} and b_{av} for normal galaxies.

I mag	a_{av} mag arcsec^{-2}	b_{av} arcsec^{-2}
<18	19.82	0.01
18-21	18.20	0.10
21-24	10.85	0.45

Values of a_{av} and b_{av} are listed in Table 6.

Following Binney and Merrifield (1998), we consider two typical kinds of galaxy, an elliptical and a spiral, each characterized by a particular brightness profile, where all considerations about spirals can be applied to lenticulars too.

For an elliptical galaxy, the brightness profile is well described by de Vaucouleurs $r^{\frac{1}{4}}$ law (Binney and Merrifield 1998, Eq. 4.18 at page 186)

$$\mu_{I,e}(r) = \mu_{I,hl,e} + 8.325 \left[\left(\frac{r}{r_{hl}} \right)^{1/4} - 1 \right] , \tag{16}$$

where $\mu_{I,hl,e}$ is the surface brightness at the half-light radius.

Using this profile, we can express $\mu_{I,hl,e}$ as a function of the galaxy I magnitude. Thus we obtain

$$\mu_{I,hl,e} = 2.5 \log(22.68 k_r^2) - 90 c_r + (5 c_r + 1) I = a_{hl,e} + b_{hl,e} I \ . \tag{17}$$

Values of $a_{hl,e}$ and $b_{hl,e}$ are listed in Table 7.

Table 7: Values of $a_{hl,e}$ and $b_{hl,e}$ for normal elliptical galaxies.

I mag	$a_{hl,e}$ mag arcsec^{-2}	$b_{hl,e}$ arcsec^{-2}
<18	21.21	0.01
18-21	19.59	0.10
21-24	12.24	0.45

For a spiral galaxy, as discussed in §4.4.3 of Binney and Merrifield (1998), the inner parts follow the same law as elliptical galaxies, while the outer parts follow the exponential law

$$\mu(r) = \mu_d + 1.086 \frac{r}{r_d} \ . \tag{18}$$

An exponential law is equivalent to de Vaucouleurs generalized $r^{\frac{1}{n}}$ law with $n = 1$ and $b_1 = 0.729$ (Binney and Merrifield 1998 Eq. 1 in Box 4.1), that is

$$\mu(r) = \mu_{hl} + 1.825 \left(\frac{r}{r_{hl}} - 1 \right) \ . \tag{19}$$

The transformation between the coefficients in Eqs. 18 and 19 is therefore

$$\mu_{hl} = \mu_d + 1.825 \ , \qquad r_{hl} = 0.595 r_d \ . \tag{20}$$

Thus, a law of the form of Eq. 16 may be used as a rough approximation to the inner parts, whereas a law with the form of Eq. 19 could be used to approximate the outer parts. The scale lengths of the two laws, so far indicated with r_{hl}, will however be different, and both will be different from the half-light radius of the composite brightness profile. The values of these two quantities may be determined as functions of the real half-light radius, if we assume a value for their ratio. Figure 4.52 by Binney and Merrifield, suggests that the scale length of the $r^{\frac{1}{4}}$ law (the bulge scale length) is half the scale length of the exponential law (the disk scale length). If then we require that the inner and the outer profile are continuously connected at the bulge scale length, one obtains the following expression for the brightness profile of a spiral galaxy

$$\mu_{I,s}(r) = \begin{cases} \mu_{I,hl,s} - 0.363 + 8.325\left[\left(\frac{r}{0.715r_{hl}}\right)^{\frac{1}{4}} - 1\right] & \text{if } r \leq 0.715r_{hl} \ , \\ \mu_{I,hl,s} + 0.549 + 1.825\left(\frac{r}{1.43r_{hl}} - 1\right) & \text{if } r \geq 0.715r_{hl} \ . \end{cases} \tag{21}$$

Proceeding like in the case of ellipticals, for the surface brightness at the half-light radius we obtain

$$\mu_{I,hl,s} = 2.5\log(15.21k_r^2) - 90c_r + (5c_r + 1)I = a_{hl,s} + b_{hl,s}I \ . \tag{22}$$

Values of $a_{hl,s}$ and $b_{hl,s}$ are listed in Table 8.

Table 8: Values of $a_{hl,s}$ and $b_{hl,s}$ for normal spiral galaxies.

I mag	$a_{hl,s}$ mag arcsec^{-2}	$b_{hl,s}$ arcsec^{-2}
<18	20.78	0.01
18-21	19.16	0.10
21-24	11.81	0.45

Table 9 reports the values of $\mu_{I,av}$, $\mu_{I,hl,e}$ and $\mu_{I,hl,s}$ for different total I magnitudes.

Table 9: Surface brightness of normal galaxies according to Eqs. 15, 17 and 22. Average inside the half-light radius, and at that radius for E and S galaxies. As discussed in Section 2, the S galaxies are five times more frequent than the E galaxies.

I mag	$\mu_{I,av}$ mag arcsec^{-2}	$\mu_{I,hl,e}$ mag arcsec^{-2}	$\mu_{I,hl,s}$ mag arcsec^{-2}
14	19.96	21.35	20.92
15	19.97	21.36	20.93
16	19.98	21.37	20.94
17	19.99	21.38	20.95
18	20.00	21.39	20.96
19	20.10	22.24	21.06
20	20.20	22.34	21.16
21	21.30	21.69	21.26
22	21.75	22.14	21.71
23	21.20	22.59	22.16
24	21.65	23.04	22.61

The values listed in the second column of Table 9 may be used to estimate the number of galaxies which could be detected in GAIA astro sky mapper. On the other hand, the values listed in the third and fourth column may be used to estimate the standard error of GAIA broadband photometry at the half-light radius of a galaxy (see Section 6). Note that, as discussed in Section 5, the model values for spiral galaxies in the fourth column are typically 0.2 mag arcsec^{-2} brighter than the observations.

We also considered an alternative model, where the brightness profile of a spiral galaxy is given by the sum of an $r^{\frac{1}{4}}$ law and an exponential law. Although physically more reasonable, this model yielded almost the same results, so that we didn't consider it further.

5 Verifications

In order to test the reliability of our model, we have compared the predicted brightness profiles of galaxies with some B band profiles of bright galaxies about $B = 9.5$-12.7 mag, taken from the literature. All conversions between B and I or μ_B and μ_I are carried out by the simple relation $B = I + 1.7$.

First, the given total B magnitude of the galaxy is transformed to I magnitude. Then r_{hl} is derived from Eq. 6, whereas $\mu_{I,hl,e}$ and $\mu_{I,hl,s}$ are obtained from Eqs. 17 and 22, respectively.

For ellipticals, we compared with Capaccioli et al. (1988). The predicted B band profile is derived from Eq. 16. When we take the average of the observations along the major and minor axes we typically find that they agree with our model within 0.2 mag arcsec^{-2} in the interval $[1 \text{ arcsec} \,|\, 4\mathrm{r_{hl}}]$. Inside a radius of 1 arcsec, the observed profiles show a clear flattening which our model doesn't describe properly, so that in this region the predicted profile is systematically brighter than the observations.

For spirals, we compared with Boroson (1981). The B band observations are typically found to agree with the composite bulge-disk brightness profile described by Eq. 21 within 0.5 mag arcsec^{-2} in the interval $[0.5r_{hl} \,|\, 2\text{-}3r_{hl}]$. In particular, the predicted brightness profile inside 0.5 r_{hl} is systematically brighter than the observations, whereas at larger radii the deviations are more evenly distributed. The predicted surface brightness at the half-light radius $\mu_{I,hl,s}$ is typically 0.2 mag arcsec^{-2} brighter than the observed value. These differences could be due partly to our use of the same colour index $B - I = 1.7$ for all galaxies, since we did not know the color indices of individual galaxies, and partly to the simplified dependance of half-light radius on magnitude in Eq. 4.

In conclusion, our model altogether predicts the surface brightness in the relevant range of radii with an accuracy of about 30%.

6 Concluding remarks

The detection and measurement of surface brightness in excess of the global sky background has been discussed in CUO_39 and values were given for the sensitivity. With revised values for the total read-out noise as given in CUO_53 we conclude that galaxies of total magnitude $I = 16.5$ mag would be detected with a $SNR = 10$ in the astro sky mapper (ASM1) in Astro-2, using an area of 4×4 arcsec^2 for the detection. This follows from their half-light radius $r_{hl} = 2.0$ arcsec and their average surface brightness inside the half-light radius $\mu_{I,av} = 20.0$ mag arcsec^{-2}. There are about 1.5 million galaxies brighter than this limit on the whole sky.

A typical bright galaxy from the RC3 (de Vaucouleurs et al. 1991) has an absolute magnitude of $M_B = -19$, according to Fig. 2 in Impey and Bothun (1997), which using our standard colour index $B - I = 1.7$ means $M_I = 20.7$. For $I = 16.5$ mag, this implies a distance modulus of $m - M = 37.2$ mag, a distance of $d = 275$ Mpc or a redshift of $z \simeq 0.05$.

The surface brightness at the half-light radius is about $\mu_I = 21.4$ mag arcsec^{-2} for an E galaxy and $\mu_I = 21.0$ mag arcsec^{-2} for an S galaxy of this magnitude. The expected photometric standard error is about 0.2 mag arcsec^{-2} in i' at the latter brightness when 100 observations are stacked from Astro-2, i.e. using 6×8 pixels/sample. Since S galaxies are five times more frequent than the E galaxies, it follows that multi-colour photometry can be obtained for the central parts of most of the 1.5 million galaxies brighter than $I \simeq 16.5$ mag.

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