Detection of planetary transits with the GAIA satellite

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Bachelor project by

Anders Johansen

Copenhagen University
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Supervisor: Erik Høg

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Introduction

After the success of the Hipparcos satellite the European Space Agency ESA has decided to go for a similar mapping mission, GAIA, as cornerstone project in this and the following decade. According to plan the GAIA satellite will be launched between years 2010 and 2012. From its position in the Lagrange point 1.5 million kilometres from Earth, GAIA will spend 5 years observing positions, spectra and colors of the billion or so brightest objects of the sky.

The scientific harvest is overwhelming. Distances and velocities of hundreds of million of stars will give a unique insight to the structure and dynamics of the Milky Way. A huge amount of information will be obtained about all sorts of objects, from small planets in the Solar System, brown dwarfs and variable stars to galaxies, quasars and distant supernovae. And not least about exoplanets, by astrometric detection of the way they influence the movements of their parent stars.

Astrometry is however not the only way GAIA can contribute to finding exoplanets. Its sensitive CCD's are capable of measuring the tiny darkening of a star's light that occur when a giant planet passes in front of it, and the photometry can in this way tell the radius of the planet. A very important parameter since this gives the mass density, which is crucial for characterising the planet.

In this "bachelor"-project I shall try to estimate how many planetary transits GAIA will detect, and look at what other observations that are needed as well in order to obtain a full set of orbital and physical parameters for the planet. This is important in order to test theories of planet formation and eventually to answer the perhaps most important question of all: Is our planet system unique or does life have physical "soil" for occuring other places in the Universe?

The project is built up as follows. Chapter 1 concerns two other ways of detecting planets, namely radial velocity measurements and astrometry. In the second chapter the theory of planetary transits is treated, and finally in the third chapter I look at the distribution of exoplanets and stars, and see how many transits GAIA is then likely to detect. In the conclusion I will discuss which methods photometry must be supplemented with.

The text is set up in Latex with references in Bibtex. Figures are made with Xfig and plots with Gnuplot. Internet sites are refered to in footnotes.

Finally I would like to mention some of my main references. The comprehensive

“Concept and Technology Study Report” [1] contains all possible information about the GAIA project. Jean Schneider’s extensive and always updated “Extrasolar Planets Encyclopedia”[1] contains information about detection methods, links to scientific papers and Internet sites and a list of all exoplanets listing orbital and physical parameters for each of them. It is warmly recommended both as starting point and inspiration later on and has been a great help throughout the project. In [2] Michael Perryman writes in about exoplanets in general. More specific texts on radial velocity measurements and planetary transits, respectively, can be found in Didier Queloz’s [3] and Penny D. Sackett’s [4] contributions to “Planets Outside the Solar System: Theory and Observations”. M. Colavita describes astrometric measurements in the same book [5], and Lattanzi et. al. [6] are concerned specifically with GAIA’s astrometric detection of exoplanets.

I would like to thank my supervisor Erik Høg as well as Michael Perryman for valuable comments and suggestions.

This version (1.3) has been updated with a section on exoplanet-related space missions (Section 2.5), the discussion of the distribution of planets (Section 3.6) has been updated with more empirical data, and similar investigations into GAIA’s ability to detect planetary transits are treated in the new Section 3.10.

[1] http://www.obspm.fr/encycl/encycl.html

Chapter 1

Radial velocity and astrometry

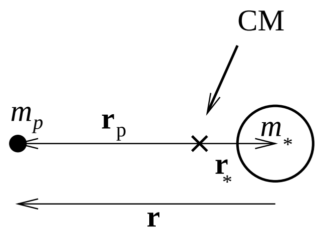
In this chapter I will describe the effect a planet has through gravity on the movement of its parent star, and ways to detect this movement. That is, with radial velocity measurements and astrometry.

1.1 Two-body problem

In a simple model a planetary system consists of one very heavy star with the mass $m_\star$ and one planet with the mass m_p. The movements of the star and the planet in this system can be obtained analytically as the classical two-body problem for two masses affecting each other only through their gravities. The solution is the equation of motion

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{G(m_\star + m_p)}{r^2}\mathbf{e}_r \,, \tag{1.1}$$

where $\mathbf{r} = \mathbf{r}_p - \mathbf{r}_\star$ denotes the radius vector from the centre of the star to the centre of the planet, and $\mathbf{e}_r$ is a unit vector in the same direction

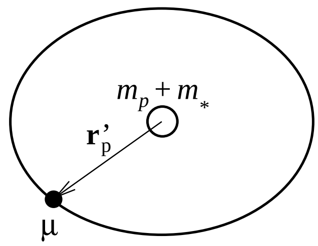


If one instead identifies $\mathbf{r}$ with the planet's coordinates in the star's (primed) rest frame, $\mathbf{r} = \mathbf{r}'_p = \mathbf{r}_p - \mathbf{r}_\star$, the equation looks a bit different,

$$\frac{d^2\mathbf{r}'_p}{dt^2} = -\frac{G(m_\star + m_p)}{{r'_p}^2}\mathbf{e}_{r'_p} \,, \tag{1.2}$$

and now obviously describes the equation of motion of a body moving in the gravitational field from a central mass $m_\star + m_p$ nailed in the origo of the system. The solution to the equation is, of course, that the orbit relative to the central mass is an ellipse.

The centre of mass is, however, not in rest in this system so it is not an inertial system. To transform the coordinates to the (unprimed) rest frame of the centre of mass, we use the definition of centre of mass in an isolated system,



$$m_\star \mathbf{r}_\star + m_p \mathbf{r_p} = 0\,. \tag{1.3}$$

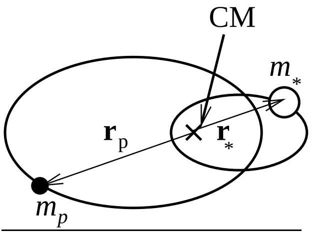
By inserting $\mathbf{r}'_p = \mathbf{r_p} - \mathbf{r}_\star$ the wanted transformation to the centre of mass system is obtained:

$$\mathbf{r}_\star = -\frac{m_p}{m_\star + m_p}\mathbf{r}'_p\,, \tag{1.4}$$

$$\mathbf{r_p} = \frac{m_\star}{m_\star + m_p}\mathbf{r}'_p\,, \tag{1.5}$$

$$\mathbf{r}_\star = -\frac{m_p}{m_\star}\mathbf{r_p}\,. \tag{1.6}$$

The first two equations show that the planet and the star both follow elliptical orbits, because r'_p describes an ellipse. The last equation states something about the relationship between these two orbits, namely that the star, due to its obligation to keep the position of the centre of mass constant, in a sense performs an opposite movement of the planet's around the centre of mass. Equation (1.6) shows that when the planet follows an ellipse orbit, the star will follow the opposite ellipse orbit with the axes scaled down by $\frac{m_p}{m_\star}$.



Planet	CM with Sun
Mercury	9.6 km
Venus	264 km
Earth	449 km
Mars	73 km
Jupiter	742000 km
Saturn	408000 km
Uranus	128000 km
Neptune	232000 km
Pluto	38 km

Distance from the Sun's centre to the centre of mass between the Sun and the planets of the Solar System.

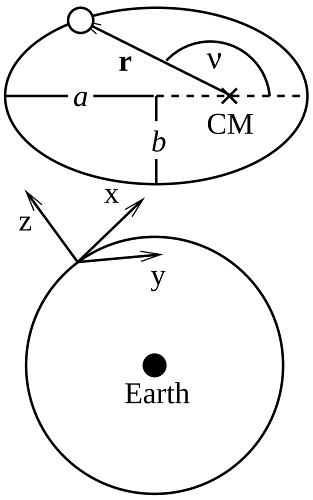
In the solar system the centre of mass between the Sun and Jupiter, which has only a weakly elliptical orbit, is situated around 750000 km from the Sun's centre, that is (astronomically speaking) just above the Sun's surface.

1.2 Projection on the line of sight

The alignment of the orbital ellipse is totally arbitrary with respect to an observer on Earth who will only be able to see its projection on the sky. With Doppler effect measurements one can, however, measure the radial component of the star's movement. Even if we can't see the spatial extent of the radial movement it will show up as periodic Doppler displacements of the spectral lines. These displacements yield an indirect way to measure the mass of a star's visible partners, such as a partner star in a binary system, or even bodies which can not be seen because of their small spatial extent or because the light drowns in the light of the parent star. That could be faint partner stars, brown dwarfs, or more interesting in this connection heavy planets.

Generally an elliptical orbit can be described in polar coordinates by

$$r(\nu) = \frac{a(1-e^2)}{1 + e\cos\nu}\,, \tag{1.7}$$



where ν is the angle between periastron (ie. where the distance to the CM in one focus is shortest) and the radius vector, a is the semimajor axis, and e is the eccentricity defined as $e = \sqrt{1 - b^2/a^2}$, here b denotes the semiminor axis. It is seen that when $a = b$ we have $e = 0$. The eccentricity of a circle is therefore 0 rising asymptotically towards 1 when the semimajor axis grows gradually bigger than the semiminor axis.

Reference [7] uses a coordinate system where the x- and y-axis span the tangent plane to the line of sight and z is the radial axis with -z pointing towards Earth.

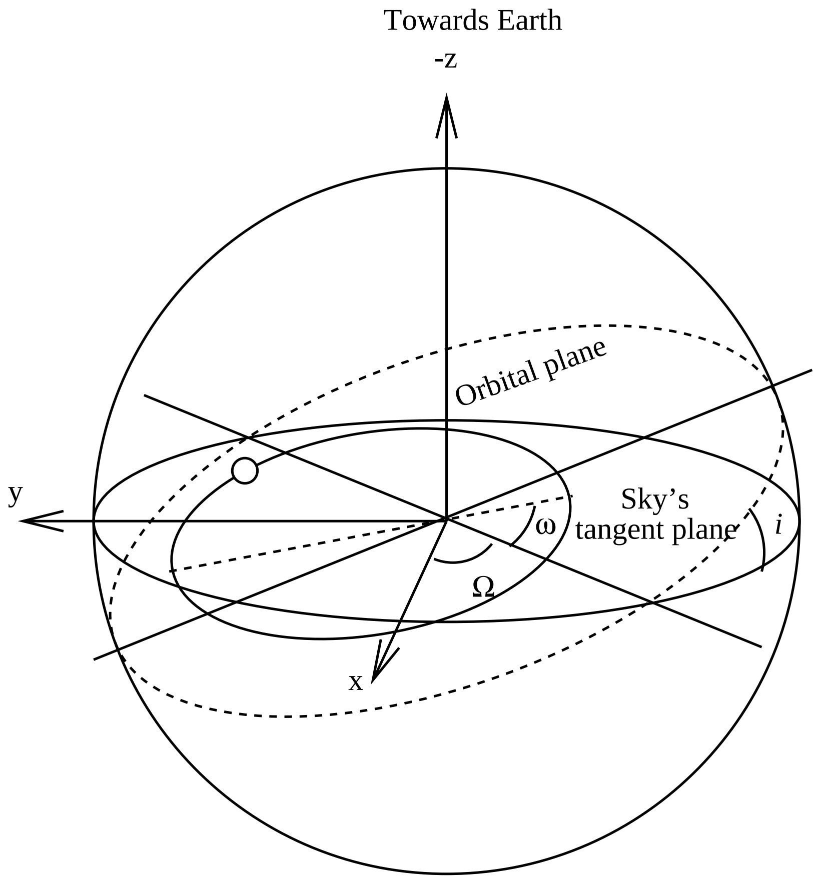


Figure 1.1: The angles in an elliptical orbit with a general orientation, see [7] page 149.

To define the orientation of the orbit we need three angles (and three parametres a, e, and ν to describe the position in the orbit), see figure 1.1:

- i and Ω are the angles between the orbital plane and the sky's tangent plane, and
- ω is the angle in the orbital plane between the intersection line of the planes and the semimajor axis of the ellipse.

The projection of the orbital ellipse on the z-axis becomes (see [7] p. 149)

$$z = r \sin(\nu + \omega) \sin i \, . \tag{1.8}$$

Now we have obtained an expression for z as a function of the orbital parameters r and ν together with the two orientation angles ω and i. It turns out that i, or rather $\sin i$, is a pathological parameter which will remain in the final expression for the mass of the planet.

1.3 Radial velocity

To find the velocity along the z-axis, we differentiate the equation for the projection (1.8) with regards to time (r and ν are time dependent),

$$\frac{dz}{dt} = \sin(\nu + \omega) \sin i \frac{dr}{dt} + r \cos(\nu + \omega) \sin i \frac{d\nu}{dt} . \tag{1.9}$$

Now we need expressions for both dr/dt and $rd\nu/dt$. The calculations are from [7] pp. 149-151. The time derative of r is obtained from (1.7),

$$\begin{aligned} \frac{dr}{dt} &= \frac{d}{dt} \left(\frac{a(1-e^2)}{1 + e \cos \nu} \right) \\ &= -\frac{a(1-e^2)}{(1 + e \cos \nu)^2} e(-\sin \nu) \frac{d\nu}{dt} \\ &= \frac{e \sin \nu}{a(1-e^2)} r^2 \frac{d\nu}{dt} . \end{aligned} \tag{1.10}$$

Kepler's well known second law for a general elliptical orbit, which states that the radius vector sweeps out equal areas in equal times, gives an expression for $d\nu/dt$:

$$r^2 \frac{d\nu}{dt} = \frac{2\pi}{P} a^2 \sqrt{1-e^2}, \tag{1.11}$$

where P is the constant period of the movement. This can now be put into (1.10);

$$\begin{aligned} \frac{dr}{dt} &= \frac{2\pi}{P} a^2 \sqrt{1-e^2} \frac{e \sin \nu}{a(1-e^2)} \\ &= \frac{2\pi}{P} a \frac{e \sin \nu}{\sqrt{1-e^2}} . \end{aligned} \tag{1.12}$$

Finally we need an expression for $rd\nu/dt$ in order to proceed from (1.9).

$$\begin{aligned} r\frac{d\nu}{dt} &= r^2 \frac{d\nu}{dt} \frac{1}{r} \\ &= \frac{2\pi}{P} a^2 \sqrt{1-e^2} \frac{(1 + e \cos \nu)}{a(1-e^2)} \\ &= \frac{2\pi}{P} a \frac{(1 + e \cos \nu)}{\sqrt{1-e^2}} . \end{aligned} \tag{1.13}$$

From all this work (1.9) now becomes

$$\begin{aligned} \frac{dz}{dt} &= \frac{2\pi}{P} a \frac{e \sin \nu}{\sqrt{1-e^2}} \sin(\nu + \omega) \sin i \\ &\quad + \frac{2\pi}{P} a \frac{(1 + e \cos \nu)}{\sqrt{1-e^2}} \cos(\nu + \omega) \sin i \\ &= \frac{2\pi}{P} \frac{a \sin i}{\sqrt{1-e^2}} \{ (e \sin(\nu + \omega) \sin \nu + (1 + e \cos \nu) \cos(\nu + \omega) \} \\ &= \frac{2\pi}{P} \frac{a \sin i}{\sqrt{1-e^2}} \{ e \left(\sin(\nu + \omega) \sin \nu + \cos(\nu + \omega) \cos \nu \right) + \cos(\nu + \omega) \} . \end{aligned}$$

The trigonometric identity $\sin\alpha\sin\beta + \cos\alpha\cos\beta = \cos(\alpha-\beta)$ finally yields

$$\frac{dz}{dt} = \frac{2\pi}{P}\frac{a\sin i}{\sqrt{1-e^2}}\left\{e\cos\omega + \cos(\nu+\omega)\right\} . \tag{1.14}$$

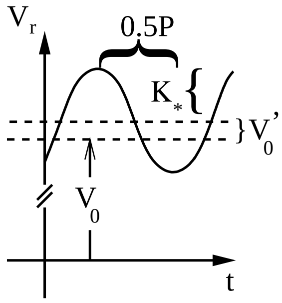
Equation (1.14) can be put on the more normal form

$$V_r = K_j(\cos(\nu+\omega) + e\cos\omega) + V_0, \tag{1.15}$$

where

$$K_j = \frac{2\pi}{P}\frac{a_j\sin i}{\sqrt{1-e^2}} \tag{1.16}$$

applies to the jth component of the system, ie. where $j=1$ denotes the star and $j=2$ denotes the planet (as in [3] p. 230), and the velocity difference between the observer and the system's centre of mass, V_0, has been added.



We see that the final expression consists of a constant term from the velocity of the centre of mass relative to Earth (of magnitude several kilometres per second) and an extra constant contribution $V_0^{'}$ (of magnitude eK_j) from the orientation and geometry of the orbit. On top of the constant part comes a periodic cosine term in the polar angle ν with amplitude K_j. For a circular orbit where $\nu \propto t$ it gives a nice harmonic oscillation in time, eccentricity on the other hand will make ν non-linear in t and lead to a "crooked" curve. The orbital parameters, including e, can however still be determined by analysing the curve further, but this is a bit technical ([8] pp. 476-477).

As an example, the amplitude in the Sun's movement due to Jupiter is 12.5 m/s, see table in margin.

1.4 Determining the mass

From Kepler's third law and the definition of centre of mass (1.3) we can find a connection between the semimajor axis of the star's orbit and the two masses ([3] pp. 230-231),

$$\left\{\begin{array}{rcl} \frac{P^2}{a^3} & = & \frac{4\pi^2}{G(m_\star+m_p)} \\ a & = & a_\star + a_p \\ m_\star a_\star & = & m_p a_p \end{array}\right\} \Rightarrow a_\star^3 = \frac{P^2G}{4\pi^2}\frac{m_p^3}{(m_\star+m_p)^2} . \tag{1.17}$$

Planet	$K_\star/\sin i$
Mercury	0.008 m/s
Venus	0.09 m/s
Earth	0.09 m/s
Mars	0.008 m/s
Jupiter	12.5 m/s
Saturn	2.8 m/s
Uranus	0.3 m/s
Neptune	0.3 m/s
Pluto	~ 0 m/s

Radial velocity amplitudes for the movement of the Sun due to interaction with the planets of the Solar System.

This expression can now be put into the expression for $K_\star$ (1.16).

$$\begin{aligned} K_\star &= \frac{2\pi}{P}\frac{\sin i}{\sqrt{1-e^2}}m_p\left(\frac{P^2G}{4\pi^2}\frac{1}{(m_\star+m_p)^2}\right)^{1/3} \Rightarrow \\ \frac{(m_p\sin i)^3}{(m_\star+m_p)^2} &= \frac{P}{2\pi G}K_\star^3(1-e^2)^{3/2} . \end{aligned} \tag{1.18}$$

The left side of the expression is called the mass function and combines the two masses together with the inclination angle. The right side can be determined observationally by measuring the radial velocity over several periods. The mass of the star can be calculated indirectly from stellar models. To a good approximation the mass of the planet in $m_\star + m_p$ vanishes in the limit $m_p \ll m_\star$. In the same limit, where also $a_p \gg a_\star$, the final expressions for the planet's mass and semimajor axis become (using Kepler's third law again):

$$
\begin{aligned}
m_p \sin i &\simeq \left(\frac{P}{2\pi G}\right)^{1/3} K_\star m_\star^{2/3} \sqrt{1-e^2} \\
a_p &\simeq \left(\frac{G}{4\pi^2}\right)^{1/3} P^{2/3} m_\star^{1/3} \,.
\end{aligned}
\tag{1.19}
$$

In this way measuring radial velocities leads to a value for the planet's semimajor axis (and eccentricity), but for the mass determination there is a degeneracy between inclination angle and mass which it is impossible to get rid of. Since i lies between $0°$ and $90°$, $\sin i$ lies between 0 and 1, and we see that $m_p \sin i$ actually expresses a minimum mass for the planet.

Star	$m_p \sin i$ [M_J]	a [AU]	P [days]	e	Star	$m_p \sin i$ [M_J]	a [AU]	P [days]	e
HD83443	0.35	0.038	2.9861	0.08	16 CygB	1.5	1.70	804	0.67
	0.16	0.174	29.83	0.42	HD 134987	1.58	0.78	260	0.25
HD 16141	0.215	0.35	75.82	0.28	HD 4203	1.64	1.09	406.0	0.53
HD 168746	0.24	0.066	6.409	0.	HD 68988	1.90	0.071	6.276	0.14
HD 46375	0.249	0.041	3.024	0.	HD 160691	1.97	1.65	743	0.62
HD 108147	0.34	0.098	10.881	0.558	HD 19994	2.0	1.3	454	0.2
HD 75289	0.42	0.046	3.51	0.054	HD 213240	3.7	1.60	759	0.31
51 Peg	**0.47**	**0.05**	**4.2293**	**0.0**	Gliese 876	1.98	0.21	61.02	0.27
BD -10 3166	0.48	0.046	3.487	0.		0.56	0.13	30.1	0.12
HD 6434	0.48	0.15	22.09	0.30	HD 92788	3.8	0.94	340	0.36
HD 187123	0.52	0.042	3.097	0.03	HD 8574	2.23	0.76	228.8	0.40
HD 209458	**0.69**	**0.045**	**3.524738**	**0.0**	HR810	2.26	0.925	320.1	0.161
υ And	0.71	0.059	4.6170	0.034	47 Uma	2.41	2.10	1095	0.096
	2.11	0.83	241.2	0.18		0.76	3.73	2594	<0.1
	4.61	2.50	1266.6	0.41	HD 23079	2.54	1.48	627.3	0.06
HD 192263	0.76	0.15	23.87	0.03	HD 12661	2.83	0.789	264.5	0.33
ϵ Eridani	0.86	3.3	2502.1	0.608	HD 169830	2.96	0.823	230.4	0.34
HD 38529	0.81	0.1293	14.41	0.280	14 Her	3.3	2.5	1619	0.3537
HD 4208	0.81	1.69	829.0	0.04	GJ 3021	3.31	0.49	133.82	0.505
HD 179949	0.84	0.045	30.093	0.05	HD 80606	3.41	0.439	111.78	0.927
55 Cnc	0.84	0.11	14.648	0.051	HD 195019	3.43	0.14	18.3	0.05
	$> 5?$	> 4	$> 2920?$	-	Gl 86	4	0.11	15.78	0.046
HD 82943	0.88	0.73	221.6	0.54	τ Boo	3.87	0.0462	3.3128	0.018
	1.63	1.16	444.6	0.41	HD 50554	4.9	2.38	1279.0	0.42
HD 121504	0.89	0.32	64.6	0.13	HD 190228	4.99	2.31	1127	0.43
HD 114783	0.9	1.20	501.0	0.10	HD 168443	7.2	0.29	57.9	0.54
HD 37124	1.04	0.585	155	0.19	HD 222582	5.4	1.35	576	0.71
HD 130322	1.08	0.088	10.724	0.048	HD 28185	5.6	1.0	385	0.06
ρ CrB	1.1	0.23	39.645	0.028	HD 178911 B	6.47	0.326	71.50	0.145
HD 52265	1.13	0.49	118.96	0.29	HD 10697	6.59	2.0	1083	0.12
HD 177830	1.28	1.00	391	0.43	70 Vir	6.6	0.43	116.6	0.4
HD 217107	1.28	0.07	7.11	0.14	HD 106252	6.81	2.61	1500.0	0.54
HD 210277	1.28	1.097	437	0.45	HD 89744	7.2	0.88	256	0.7
HD 142	1.36	0.980	338.0	0.37	HD 33636	7.71	2.62	1553.0	0.39
HD 27442	1.43	1.18	423	0.02	HD 39091	10.37	3.34	2115.3	0.62
HD 74156	1.56	0.276	51.61	0.649	HD141937	9.7	1.49	658.8	0.40
	> 7.5	4.47	2300.0	0.395	HD 114762	11	0.3	84.03	0.334

Table 1.1: List of the 74 planets currently (november 2001) detected with radial velocity measurements, from Extrasolar Planets Encyclopedia. The limit between planet and brown dwarf is set at a minimum mass of 13 M_J.

1.5 Detection methods

A 10-15 m/s precision on radial velocity measurements requires a spectral resolution down to m magnitude:

$$
\frac{\Delta V}{c} = \frac{\Delta \nu}{\nu} \Rightarrow \Delta \nu = \frac{\Delta V}{c} \nu \,. \tag{1.20}
$$

Such an exquisite resolution is a real technical challenge and is obtained by observing thousands of spectral lines at the same time and constantly calibrate with a well known reference spectrum superimposed. ([3] pp. 236-241). Using such a spectrograph, namely the ELODIE [9] (which uses thorium as a reference spectrum) on a telescope in Haute de Provence, the two Université de Genevè astronomers Michel Mayor and Didier Queloz found a planet around the star 51 Peg [10] in 1995.

The planet turned out to be completely different from anything known from our solar system, with a minimum mass of 0.47 Jupiter masses, a period of only 4.2 days and a semimajor axis of 1/20 of an astronomical unit. It can be clearly seen in Table 1.1 that several planets have the same kind of orbits as 51 Peg and this has lead to the establishment of a whole new class of planets called "51 Peg"-like or Hot Jupiters.

1.6 Astrometry

Very sensitive observations of star positions open up a whole new option for detecting exoplanets. The general line of thought is the same as for the radial velocity method. Under the gravitational influence of its planets a star will go into an elliptical orbit around the system's centre of mass. The difference is that we now look at the projection of the movement on the sky, and we shall find that the extra dimension thus granted gives a certain determination of the inclination angle i and breaks the degeneracy between this and the mass of the planet.

1.7 The astrometric signature

From (1.6) we know the connection between the star's and the planet's radius vector from the system's centre of mass,

$$\begin{aligned} \mathbf{r}_\star &= -\frac{m_p}{m_\star}\mathbf{r_p} \Rightarrow \\ r_\star &= \frac{m_p}{m_\star} r_p \, . \end{aligned} \tag{1.21}$$

The star's angular change on the plane of the sky α is given by $r_\star = \alpha d$, where d is the distance from the observer on Earth to the star. This gives

$$\alpha = \frac{m_p}{m_\star}\frac{r_p}{d} \, . \tag{1.22}$$

The maximum value of α during the star's orbital movement is the astrometric signature (see [6]). If r_p is in astronomical units and d is in parsecs, then α is given in arcseconds. With very precise astrometric measurements the projection of the orbital ellipse on the plane of sky can be determined. Astrometry is especially sensitive towards planets with big orbital radii (and therefore periods) and a big mass ratio between planet and star. Contrary to the radial velocity method the distance now also matters; the greater the distance the smaller the signature. The radial velocity method is only sensitive to distance through the fact that there have to be enough photons, so the further away the star is the more luminous it has to be.

Planet	α
Mercury	0.006 μas
Venus	0.18 μas
Earth	0.30 μas
Mars	0.049 μas
Jupiter	497 μas
Saturn	273 μas
Uranus	83.7 μas
Neptune	156 μas
Pluto	0.026 μas
51 Peg	2.23 μas

Astrometric signature for planets in the Solar System at a distance of 10 parsecs, and for 51 Peg b at the same distance.

1.8 GAIA and astrometry

The connection between the projected orbital ellipse and the actual orbit is not trivial. The so-called Thiele-Innes elements, which are geometric places that can be determined on the projected orbital ellipse, are used to transform back to the three-dimensional orbit, see [11] pp. 156-163. They are functions of the three unkown angles and the semimajor axis of the star's orbit. Knowings the Thiele-Innes elements enables a determination of ω, Ω and more interesting i and $a_\star$. The eccentricity of the orbit can also be determined from the projected orbital ellipse, see [11] pp. 154-156.

When the semimajor axis and period of the star are determined, and the mass of the star known from star models, the important parameters mass and semimajor axis of the planet can be determined the same way as for the radial velocity method,

$$\begin{aligned} a_\star^3 &= \frac{P^2 G}{4\pi^2}\frac{m_p^3}{(m_\star + m_p)^2} \simeq \frac{P^2 G}{4\pi^2}\frac{m_p^3}{m_\star^2} \Rightarrow \\ m_p &= a_\star m_\star^{2/3}\left(\frac{4\pi^2}{P^2 G}\right)^{1/3} , \\ a_p &= \frac{m_\star}{m_p} a_\star \, . \end{aligned} \tag{1.23}$$

One of GAIA's primary objectives is to measure star positions very precisely, and the satellite is therefore very suitable for finding planets using astrometry. With a precision of down to 10μas, it will be able to find a great percentage of giant planets in Jupiter-like orbits within $\sim$200 pc (see [6] figure 1-2). Using the detection probabilities calculated in [6], it is estimated in [12] that GAIA will detect between 8000 and 57000 Jupiter mass planets within our galaxy.

One of the great advantages of astrometry is that only partner objects can cause the movement, since variability and pulsations in the star can't cause a displacement of the photocentre of magnitude 10μas level according to [6]. Astrometry is most sensitive to long periods, contrary to radial velocity measurements, and it is therefore an obvious way to find planets with periods of several years. With a GAIA mission lifetime of 5 years there is hope to find planets with similar or longer periods.

There is not much hope to be able to use astrometry to pin the inclination angle in systems found by radial velocity measurements. As an example from the table in the margin we see that 51 Peg b gives only 2.2 μas at 10 parsec (the real distance to 51 Peg is 13.6 pc). The problem is the small orbital radius.

Chapter 2

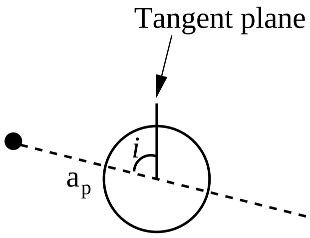
Planetary transits

When a planet passes the disc of its star, it blocks a small percentage of the star's light. Sensitive photometry can tell the characteristic dip in the light curve, caused by the planet. But good equipment is not enough — the inclination angle of the planet's orbit must lie in a very narrow window to bring the planet to pass between Earth and the star. So either one has to rely on pure luck to observe a planetary transit. But even better is to observe a lot of stars, and in that way multiply the small probability for a single transit by hundreds or thousands of observed stars. Or in the case of GAIA several hundred millions of stars.

Planetary transits are the only one of the methods discussed in this project that can be used to determine the radius of the blocking planet. Together with the mass this yields the important parameter *mass density*.

2.1 Geometric probability

In this section I will try to quantify the intuitive fact that planetary transits can not be detected around all stars. I assume that the inclination angle i between the planet's plane of orbit and the sky's tangent plane is randomly distributed between 0° and 90°, that planetary orbits are circular and that the surface brightness, mass and radius of the planet can be neglected compared to the parent star. I also assume that the orbital radius is much greater than the radius of the star. The theory in this chapter is based on [4].



We see from the figure in the margin that the inclination angle must obey

$$a_p \cos i \leq R_\star + R_p \,. \tag{2.1}$$

If $\cos i$ does not fulfil this condition we can not observe any transit. The condition is only one on the position of the observer and not a physical discrimination against certain inclination angles. For Jupiter this implies that the inclination angle must be higher than 89.94°! This is indeed a strict demand, considering as mentioned that i is randomly distributed between 0° and 90°. If the orbital radius of Jupiter was 0.05 AU, as for 51 Peg b, the inclination angle would only be required to be higher than 84°.

Planet	P_{geom}
Mercury	1.15 %
Venus	0.62 %
Earth	0.45 %
Mars	0.29 %
Jupiter	0.094 %
Saturn	0.051 %
Uranus	0.024 %
Neptune	0.015 %
Pluto	0.011 %
51 Peg b	9.95 %

Geometric transit probability for planets in the Solar system and 51 Peg b around 51 Peg.

The probability that a planet (or rather, the observer) even fulfil (2.1) is

$$P_{geom} = \frac{\int_0^{(R_\star+R_p)/a_p} d(\cos i)}{\int_0^1 d(\cos i)} = \frac{R_\star + R_p}{a_p} \simeq \frac{R_\star}{a_p} . \tag{2.2}$$

The last approximation is good if the radius of the planet is very small compared to the star, while big planets such as gas giants increase the transit probability with their great radii. For 51 Peg b, the first exoplanet detected around a main sequence star, the probability is almost 10 % (I have set the star's radius to be the same as the Sun's because 51 Peg is of spectral type G2, the planet's radius to 1.3 Jupiter radii [13] and its orbital radius to 0.051 AU). 51 Peg b with its great mass and small orbital radius is a typical example of the new class of planets that radial velociry measurements have found. There has been searched for periodic darkening from 51 Peg b in the light curve of 51 Peg [14], but since there has not been detected any this suggests that the inclination angle is too small. Under all circumstances the lack of transits puts an upper limit on the inclination angle to 85°.

One should think that if the orbit of a single planet is aligned correctly other possible planets in the system will follow automatically. But unfortunately, in the Solar System the few degrees of difference between the planets's orbital planes is enough to make transits from more than one planet unlikely, according to [4].

Planet	Max $\frac{\lvert\delta F_\lambda\rvert}{F_\lambda}$
Mercury	0.001 %
Venus	0.008 %
Earth	0.008 %
Mars	0.002 %
Jupiter	1.06 %
Saturn	0.75 %
Uranus	0.13 %
Neptune	0.13 %
Pluto	0.0003 %
51 Peg b	1.8 %

Maximum flux reduction for planets in the Solar System and for 51 Peg b around 51 Peg.

2.2 Flux reduction

One of the great advantages of a planetary transit is that it can give us knowledge about the radius of the planet. If we observe a star with a constant flux density at the surface $F_{\lambda,\star}$ at a certain wavelength λ become darkened by a planet with flux density $F_{\lambda,p}$ (on its night side), the maximum flux reduction will occur when the whole planet is inside the stellar disc. Before the transit the total flux is

$$F_\lambda = \pi F_{\lambda,\star} R_\star^2 + \pi F_{\lambda,p} R_p^2 . \tag{2.3}$$

The planet will block a part of the star's light, and the relative reduction is

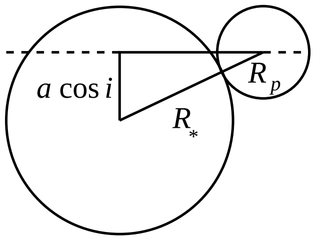
$$\mathrm{Max}\frac{|\delta F_\lambda|}{F_\lambda} = \frac{\pi F_{\lambda,\star} R_p^2}{\pi F_{\lambda,\star} R_\star^2 + \pi F_{\lambda,p} R_p^2} \simeq \left(\frac{R_p}{R_\star}\right)^2 , \tag{2.4}$$

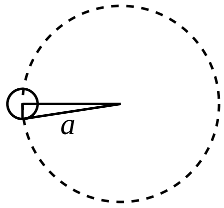
when the planet's flux density (still on the night side) is much smaller than the star's, a very good approximation. Jupiter will with its radius of 10 % of the Sun's cause a flux reduction of around 1 %.

2.3 Duration

The other crucial factor for detecting a planetary transit is the duration. Jupiter will in its 11.9 years orbit only spend a small fraction of its orbit transiting the Sun's disc, for a given observer on an exoplanet from which Jupiter's inclination angle satisfies the condition for a transit to occur. The duration of the transit t_T is defined as the period of time in which the star's luminosity is affected geometrically by the transiting planet,

$$t_T \equiv P\frac{1}{2\pi}2\arcsin\left(\frac{\sqrt{(R_\star + R_p)^2 - a_p^2\cos^2 i}}{a_p}\right) , \tag{2.5}$$





which for $a \gg R_\star \gg R_p$ becomes

$$t_T = \frac{P}{\pi}\sqrt{\left(\frac{R_\star}{a_p}\right)^2 - \cos^2 i} \leq \frac{PR_\star}{\pi a_p} . \tag{2.6}$$

Since $P^2 \propto a_p^3$, $t_T \propto \sqrt{a}$. During the 4350 days of Jupiter's orbit only maximally 1.23 days are spent transiting the Sun (seen from the same "lucky" exoplanet as before). This is an additional depreciation of the detection probability, the fact that not only do you have to be at the right place relative to a given star, but also you must observe it at the right time. This can of course be overcome by observing a star for the entire duration of the planet's orbit, but without certain knowledge about whether a transit can occur at all one has to observe a number of stars for up to several years.

Planet	t_T
Mercury	0.33 d
Venus	0.46 d
Earth	0.54 d
Mars	0.67 d
Jupiter	1.23 d
Saturn	1.66 d
Uranus	2.36 d
Neptune	2.94 d
Pluto	3.39 d
51 Peg b	0.12 d

Maximum transit time for planets in the Solar System and for 51 Peg b.

In 1999 astronomers Tim Brown and David Charbonneau observed the star HD 209458. Radial velocity measurements had earlier revealed an exoplanet in a 51 Peg-like orbit around the star. This led to the first detection of a planetary transit [15] and with that a value for the inclination angle of the orbit and the planet's radius. The transit has been detected several times since then. The most precise measurements were performed with the Hubble Space Telescope [16]. This led to an inclination of $i = 86.6°$ and therefore a mass of 0.69 M_{Jup} (from the radial velocity measurements). Combined with the observed radius of 1.347 R_{Jup} times Jupiter's (or around 94000 kilometres) the planet has a density of around 0.35 g/cm^3, which is the first indication that 51 Peg planets are indeed gas planets and not gigantic terristrial planets. Even for a gas giant the density is low, but this is probably due to its close proximity of only a few stellar radii to its parent star. The high temperature has "inflated" the planet's atmosphere [17].

2.4 Limb darkening

Equation (2.4) gives a value for the radius of the blocking planet when the radius of the star is known. The limb darkening of the star has, however, not been considered. If the planet passes through the limb darkened edge of the stellar disc, (2.4) will yield a smaller value for the flux reduction than for a planet passing through a smaller amount of limb darkening at the middle of the star's disc. This will lead to an underestimate of the planet's radius [4]. [18] gives for HD 209458 a value of $c_\lambda = 0.56$ in the simple limb darkening model $B_\lambda(\mu) = 1 - c_\lambda(1-\mu) < 1 - c_\lambda$, so it is a great underestimate of the planet's radius that can result. Since mass density is proportional to the third power of the radius, the resulting error in mass density can be severe.

A correct model of the limb darkening and knowledge of the inclination angle are therefore essential for obtaining the radius of the planet. The inclination angle can be found by inverting the first equation in (2.6). That requires knowledge of the duration of the transit and the period of the planet's orbit, the semimajor axis follows from Kepler's third law. Determining t_T and P is not a problem if one follows the star's light curve over a long time and with several observations every day. The problem is that GAIA doesn't do that. In its 5 year lifetime GAIA observes all objects at least 30 times, 4 times each time [6] [1]. It is not very likely to determine both t_T or P in 30 random observations. But photometric follow-up from telescopes on the ground or photometric space telescopes can determine the light curve in details. GAIA's observation has then shown that the planet's orbit is aligned correctly for a transit to occur, which is not a small contribution considering the geometric and time probabilities.

Another way to determine the inclination angle is astrometry, but that is only good for big orbital radii. Radial velocity measurements give no value for the inclination, but if a transit has occured the mass can be determined since $\sin i$ can be safely set to 1.

2.5 Space missions

Excellent photometric conditions are obtainable from space observations. The effects of scintillation due to the atmosphere, clouds, and sub-optimal seeing are effectively avoided by sending one's observatory into space. GAIA will be preceded by numerous photometric space missions. Small missions such as the Canadian MOST and the Danish MONS will be launched during the next few years, and will among other things look for planet transits from giant planets. France/ESA project COROT will be launched in 2004 and be able to detect transiting planets slightly bigger than Earth.

Two giants are the already selected NASA mission Kepler (launch: 2005), and the ESA candidate transit mission Eddington which is expected to detect several thousand giant planets and 2000 terristrial size planets as well.[1]

[1] See Extrasolar Planets Encyclopedia for full list of future exoplanet-related space missions.

Chapter 3

Model of planetary transits

In this chapter I try to estimate how many transiting planets GAIA is likely to detect. That requires a model of how planets are distributed and knowledge about GAIA's photometric ability. I use much of the theory of planetary transits from Chapter 2 also.

3.1 Model and parameters

The final number of transiting planets depends on both geometric probabilities, the distribution of stars and planets in the Milky Way and the question of which star types are not so variable that it drowns the effect of a transiting planet, or makes any transit-like observation highly doubtful.

I have chosen to sculpt the model as a sum over different star classes $\star$. For a planet class p with matching orbital parameters P and a, the number of detected transits involving this class can be expressed as

$$n_p = \sum_{\star} N_\star \cdot P_{geom} \cdot P_{time} \cdot f_p \cdot V_\star \,, \qquad (3.1)$$

where

- $N_\star$ is the number of stars in class $\star$ which GAIA can detect well enough to observe a transiting planet of class p,
- P_{geom} is the geometric probability that the orbit is aligned correctly,
- P_{time} is the probability that GAIA observes a planet whose orbit is aligned correctly at the correct time,
- f_p is the frequency of the planet class p around the star class $\star$, and
- $V_\star$ is the share of stars of class $\star$ that is constant in millimagnitudes.

In the following I shall examine the parameters one by one and divide into different star and planet classes. But before that I will look at the precision of GAIA's photometry.

3.2 Precision of GAIA photometry

GAIA does photometry in the broad G band, which resembles the well-known V band ([1] 6.4.2). As shown in Chapter 2, a planet of Jupiter size causes a maximum relative flux change of about 1 % during the transit. The connection between relative flux change $\delta F/F$ and magnitude change δG is obtained by consecutive Taylor expansions in $\delta F/F \ll 1$

$$\begin{aligned}
\frac{F}{F-\delta F} &= \frac{1}{1-\delta F/F} = 10^{-0.4(G_0-G_T)} \simeq 1+\frac{\delta F}{F} \\
\Rightarrow G_T &\simeq G_0 + 2.5\log(1+\frac{\delta F}{F}) = G_0 + (2.5\log e)\ln(1+\frac{\delta F}{F}) \\
&\simeq G_0 + (2.5\log e)\frac{\delta F}{F} \simeq G_0 + 1.086\frac{\delta F}{F} \\
&\simeq G_0 + \frac{\delta F}{F}\,. \qquad (3.2)
\end{aligned}$$

Simply put, if a star's light is reduced by a certain small percentage the magnitude increases by the same percentage, only added.

Defining that the signal-to-noise ratio S/N must be at least 10 in order for an observation to be convincing, we obtain the following for the limiting magnitude change L that GAIA must be able to detect, under an increase of magnitude $\Delta G = 0.01$,

$$\begin{aligned}
\Delta G &= \frac{S}{N} L \Rightarrow \\
L &= \frac{\Delta G}{S/N} = \frac{0.01\text{mag}}{10} = 0.001\text{mag}\,. \qquad (3.3)
\end{aligned}$$

The question now is, what magnitudes GAIA can observe with one millimagnitude precision. The uncertainty on the number of electrons arriving at a CCD is given by the square root law, if one assumes a Poisson distribution. At $N_e = 10^6$ we have $\sqrt{N_e}/N_e = 0.001$. GAIA has 16 CCDs so each CCD must receive at least 62500 electrons. This happens at $G = 14.3$mag, see [1] Table 8.1 for number of electrons.

The precision becomes even better due to the fact that GAIA's observation consists of 4 measurements within approximately 4 hours [1]. Each measurement now only has to contribute one fourth of the flux. The limit magnitude rises accordingly since

$$\begin{aligned}
\frac{F}{1/4F} &= 4 = 10^{-0.4(G_0-G_{1/4})} \Rightarrow \\
G_{1/4} &= G_0 + 2.5\log 4 = G_0 + 1.5 = 14.3 + 1.5 = 15.8
\end{aligned} \qquad (3.4)$$

I choose to be a bit conservative and set the limit magnitude to be $G = 15$.

3.3 Geometric and time probability

The geometric probability is calculated in (2.2), $P_{geom} = R_\star/a$, and is connected with both star and planet class.

Since a planet whose orbit is aligned correctly for a transit to occur only spends a short fraction of its orbit in front of the star, it is not certain that even one of GAIA's measurements will detect the transit. Denoting k for the number of independent GAIA observations of a star, the probability that a transit is detected is

$$P_{time} = 1 - [1 - \frac{t_T}{P}]^k \,. \tag{3.5}$$

Here t_T is the time spent in transit and P is the period of the orbit. The highly unlikely event that all k observations occur during transit, and GAIA therefore does not detect the intensity difference, has been neglected. The ratio $\frac{t_T}{P}$, which is the percentage of its time the planet spends in the transit, can be written as a function of more basic parameters (I have used the approximation (2.6) and neglected the $\cos i$ factor):

$$\frac{t_T}{P} = \frac{PR_\star}{\pi a} \cdot \frac{1}{P} = \frac{R_\star}{\pi a} \,. \tag{3.6}$$

A (detectable!) transit has a duration of magnitude 1 day, ie. 24 hours. Each group of observations lasts 4 hours. I will therefore say that k is 30 and not 4 times as many. The important thing is that the first of the four observations detect the transit, this covers the entire duration of the transit. The other 3 will then, except towards the end of the transit, also detect the minimum. Of course it would be interesting if all 4 observations fall just as the planet passes the edge of the stellar disc and thereby reveal the shape of the light curve (and hence the inclination angle).

All in all the time probability becomes

$$P_{time} = 1 - [1 - \frac{R_\star}{\pi a}]^{30} \tag{3.7}$$

3.4 Variability

The detection of a planetary transit is very dependent on whether the star is constant on millimagnitude level. Variability is closely connected to magnetic activity which is again connected to rotation speed. A fastly rotating star has a greater degree of differential rotation towards the centre, and the friction between the different layers is like a dynamo creating magnetic fields. Magnetic fields are responsible for star spots and coronal activity, the causes of variability in main sequence stars.

Ionised calcium, Ca II, can be used as an indicator of atmospheric activity. The idea is that the flux in Ca II's H and K lines reflect a non-thermic heating of calcium in the chromosphere, ie. a heating due to the magnetic activity on the surface. The parameter R'_{HK} is defined as the ratio between the flux in the core of Ca II's H and K lines from the chromosphere and the star's bolometric flux in the same area, and the size of R'_{HK} reflects the star's magnetic activity [19].

Main sequence stars lose their angular momentum through interaction between their wind and magnetic field and gradually slow down [20]. Old stars can therefore be expected to have less magnetic activity than young stars. Early stars never brake down, and for that reason they are, according to [3], very magnetically active. That goes for spectral classes earlier than F5, see [3] and [2]. (Another reason that radial velocity measurements are reserved for F, G and K stars is that the high rotation speed of early stars Doppler widen the spectral lines. The lines combine and become wide and impossible to tell from each other.)

Variability takes place at many frequencies — from a few hours for convection phenomena to several days corresponding to the rotation period of the star for star spots and up to several years for the magnetic cycle. Variability due to a transit has a time scale of around 1 day and it affected mostly by convection phenomena (in slowly rotating stars at least), see Internet pages in footnotes on this and the next page.

NASA's Kepler mission will detect transits from Earth size planets from its orbit in space. On their homepage[1] they give an estimate of the percentage of late stars that are slowly rotating and therefore photometrically stable on a time scale of 1 day, the typical duration of a transit. They estimate that 75 % of all F, G and K stars are older than 1 to 2 billion years, and have braked down enough for magnetic variability not to coalesce with a transit from an Earth-like planet. That means more than enough to detect a Jupiter size planet which gives a hundred times greater flux change. Interestingly they write that for the remaining 25 % time can be spent in the beginning of the mission mapping the variability which can later be filtered away. According to the plan, Kepler will be launched in 2006 and contribute with important experience about large scale detection of planetary transits. The COROT satellite, a France/ESA project with scheduled launch in 2004, will from an Earth orbit among other things look for transits from planets of size down to slightly bigger than Earth. On the mission's homepage[2] they have some considerations about how many stars are photometrically stable.

3.5 Distribution on star classes

For each star class three parameters are needed: the radius of the star $R_\star$, the number of stars in the class $N_\star$ and the percentage that are photometrically stable $V_\star$.

I choose to divide the stars in the model by spectral classes: O, A, B, F, G, K and M. [1] Table 6.4 contains an estimate of how many stars GAIA can detect in the different spectral classes. For typical radii I have for each star class used the radius of sub type V in the spectral class (from [21] Table 3.13). This is of

[1] http://www.kepler.arc.nasa.gov/variability.html
[2] http://www.astrsp-mrs.fr/projets/corot/planets.html

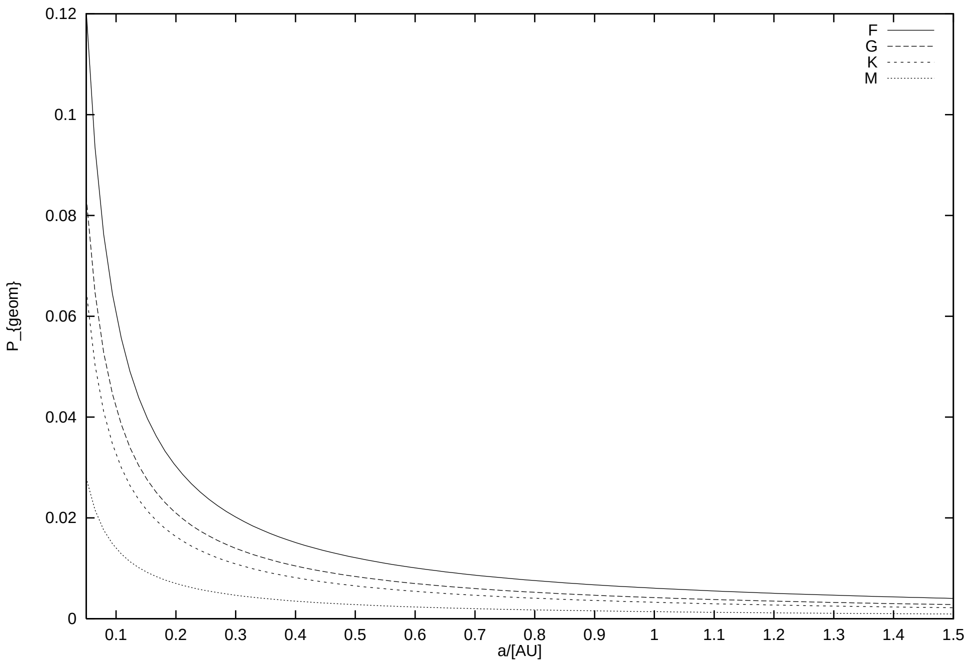


Figure 3.1: Geometric probability for the four star classes as a function of orbit radius.

course an approximation, but a useful one for the average radius of stars in a given spectral class.

For $V_\star$ I base my numbers a bit conservatively on the numbers given by the Kepler mission. That is 75 % for F stars and later (I extrapolate to be valid for M stars as well), and 0 % for earlier stars. This given 4 significant star classes, summarised in Table 4.1. Experience and data from earlier photometric space missions may make transit detecting around some inherently variable stars possible, effectively raising $V_\star$.

Class	$N_\star$	$R_\star$	$V_\star$
F	$6 \cdot 10^6$	1.3	0.75
G	$8 \cdot 10^6$	0.9	0.75
K	$9 \cdot 10^6$	0.7	0.75
M	$5 \cdot 10^5$	0.3	0.75

Table 3.1: Parameters for the four star classes in the model.

Figures 3.1 and 3.2 show plots of respectively geometric and time probability for the four star classes.

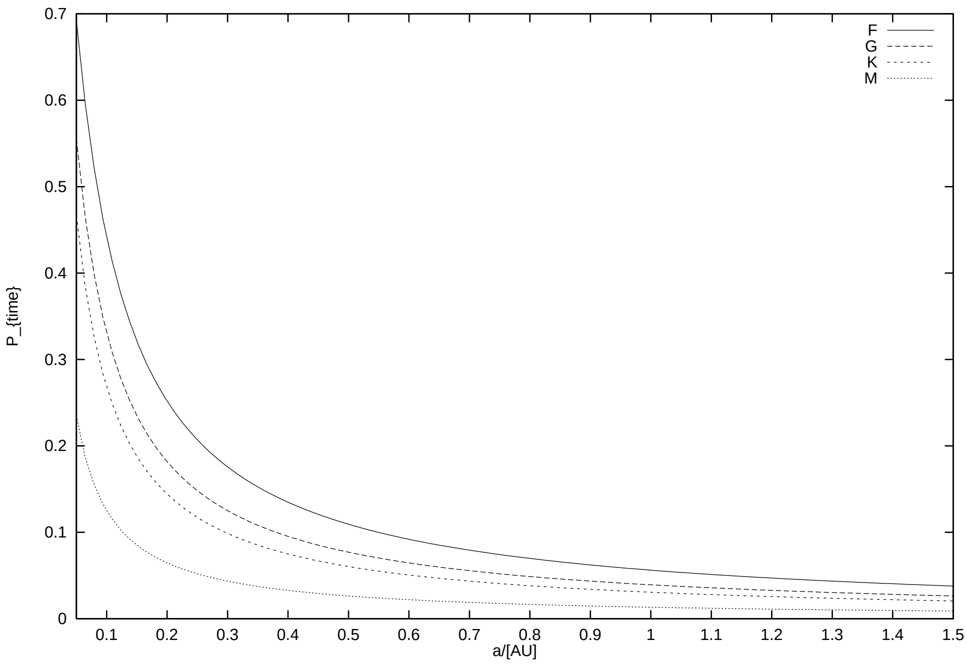


Figure 3.2: Time probability for the four star classes as a function of orbit radius.

3.6 Distribution of planets

The number of detected planetary transits depends also on how many giant planets are hiding in the dark, and their distribution. The crucial parameter, apart from the sum, is the radius of the orbit. If the majority of the giant planets hide far away from their parent stars, GAIA will never be able to catch them transiting, because the geometric and time probabilities both are small. Conversely a large population of planets in close orbits will lead to many transits.

There are two things to navigate after in estimating the distribution of planets. Firstly we know the Solar System where the gas giants are distant from the Sun, but we also know the distribution of the exoplanets so far detected. The latter is unfortunately affected by a tendency in the observation method for planets with small orbital radii.

But for small orbital radii more than 80 planets are known (at the time of writing). In Figure 3.3 I have made a histogram plot of the distribution of exoplanets in intervals of 0.1 AU. The 51 Peg-like planets can be seen clearly crowding around small a's. But the distribution continues more or less smoothly out to 2 AU and further, and I have only included the 63 planets whose semi-major axis is shorter than 2 AU. The other 21 planets have greater orbit radii.

In my model of planetary distribution I will for small orbital radii use the dis-

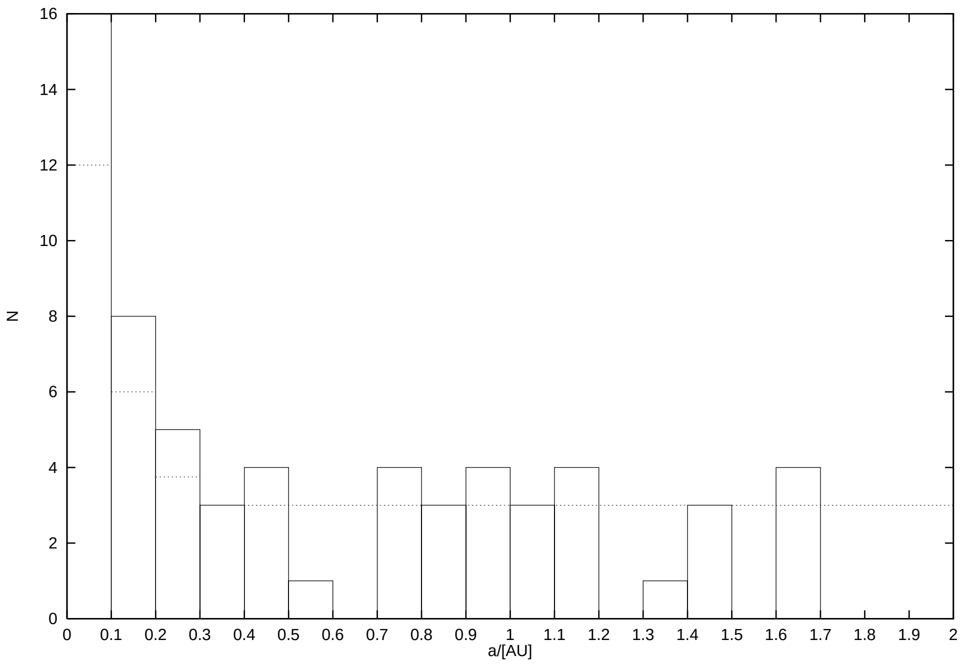


Figure 3.3: Distribution of known exoplanets with orbital semimajor axis less than 2 AU. Numbers are from Table 1.1. The dotted lines represent the distribution of planets of the model.

tribution of the already known exoplanets. Today we know from radial velocity surveys that around 5 % of Solar type stars have giant planets orbiting within 3 A.U. [2]. The internal distribution among the inner most of those 5 % can be seen in Figure 3.3. I shall look at that in the next section. After that I will give a cautious guess at the distribution further out and see what can be expected of GAIA out there.

Astronomers on the OGLE (Optical Gravitational Lensing Experiment) project recently photometrically monitored 5 million stars towards the Galactic centre on 32 nights during a time interval of 45 days [22]. Out of these, 52000 were Galactic disk stars with photometry better than 1.5 %. These were examined for flat-bottomed eclipses with the depth smaller than 0.08 mag. This revealed 46 stars with transiting low-luminosity objects. The transiting objects may be Jupiters, brown dwarfs, or M dwarfs. Although a big percentage of the stars may be intrinsically variable early type stars, and some planets may have escaped detection because of the discontinous observing periods, there is still very few transits. Assuming an average geometric transit probability of 5 %, around ≈ 2 % of the 52000 stars had transiting objects.

A more depressing result came from the Hubble Space Telescope's photometric monitoring of 34000 main sequence stars in the globular cluser 47 Tucanae [23]. Not one single light curve could be convincingly interpreted as a planet. This means that the planetary frequency in 47 Tuc is at least an order of magnitude lower than in the Solar neighbourhood. But the low metallicity and the short distances between the stars of 47 Tuc may explain the absence.

From this perspective a planetary frequency of 5 % may seem a high average, considering the evidence for geographic differences. But in the present model I shall assume 5 % anyway, and just add that all results scale with this number, and therefore a smaller frequency is easily incorporated.

3.7 Present distribution of exoplanets

As mentioned, around 5 % of late stars in the Solar neighbourhood are known to have massive planets in close orbits, and this average will be assumed to apply to all the stars GAIA will monitor.

The histogram of orbit radii is rather "lumpy". We see an accumulation of planets around small a, but further out we also see crowded regions with many planets interspaced with regions that are almost empty.

I will take the liberty to "smooth out" the distribution, and not discuss further whether the lumpy distribution is due to a real preference for certain orbital radii or a small set of data that has does not yet follow its underlying distribution. Another thing is that the $K_\star$ amplitude (in radial velocity measurements) is proportional to $1/\sqrt{a}$. One of the reasons for the accumulation towards small a is a tendency in the detection method. Even with that correction in mind, the small a are still over-represented. The small a are important since the geometric and time probabilies are high there and at the same time change rapidly.

I will in the following smooth out the distant intervals to 3 planets per 0.1 AU. The closest three intervals will be treated individually, with four times more planets in the closest (compared to the 3 further out), twice is many in the next, and 1.25 as many in the third interval.

The 3 planets out of 84 is around 0.04. Only 5 % of the stars were represented, so this gives $0.05 \cdot 0.04 = 0.002$ in real frequency. For the three close intervals we have 0.008, 0.004, 0.0025 and then 0.002 from there and out to 2 AU.

3.8 Calculations

To help with the calculations of (3.1) I have made a computer program in Perl. For each star class, the program calculates the contents of the sum in (3.1) for increasing orbital radius and stops when the increase per interval is less than 1 detected planet. Finally the program adds up the contributions per interval from each star class. The source code is in the appendix[3]. Comments in the code explains how it works. The result of running it with the planet distribution discussed in the last section is given in Table 3.2.

[3]Can be downloaded at http://www.astro.ku.dk/~ajohan/projects/bp/model/model.pl

Star cl. F (N_s=6000000 V_s=0.75 R_s=1.3)

i	a	f_p	p_geom	p_time	n_p	sum
0	0.05	0.0080	0.12096	0.69210	3013.8	3014
1	0.15	0.0040	0.04032	0.32128	233.2	3247
2	0.25	0.0025	0.02419	0.20699	56.3	3303
3	0.35	0.0020	0.01728	0.15251	23.7	3327
4	0.45	0.0020	0.01344	0.12070	14.6	3342
5	0.55	0.0020	0.01100	0.09985	9.9	3352
6	0.65	0.0020	0.00930	0.08514	7.1	3359
7	0.75	0.0020	0.00806	0.07421	5.4	3364
8	0.85	0.0020	0.00712	0.06576	4.2	3368
9	0.95	0.0020	0.00637	0.05904	3.4	3372
10	1.05	0.0020	0.00576	0.05357	2.8	3374
11	1.15	0.0020	0.00526	0.04902	2.3	3377
12	1.25	0.0020	0.00484	0.04519	2.0	3379
13	1.35	0.0020	0.00448	0.04191	1.7	3380
14	1.45	0.0020	0.00417	0.03908	1.5	3382
15	1.55	0.0020	0.00390	0.03660	1.3	3383
16	1.65	0.0020	0.00367	0.03442	1.1	3384
17	1.75	0.0020	0.00346	0.03248	1.0	3385
18	1.85	0.0020	0.00327	0.03075	0.9	3386

Star cl. G (N_s=8000000 V_s=0.75 R_s=0.9)

i	a	f_p	p_geom	p_time	n_p	sum
0	0.05	0.0080	0.08374	0.55539	2232.5	2233
1	0.15	0.0040	0.02791	0.23491	157.4	2390
2	0.25	0.0025	0.01675	0.14817	37.2	2427
3	0.35	0.0020	0.01196	0.10815	15.5	2443
4	0.45	0.0020	0.00930	0.08514	9.5	2452
5	0.55	0.0020	0.00761	0.07020	6.4	2459
6	0.65	0.0020	0.00644	0.05972	4.6	2463
7	0.75	0.0020	0.00558	0.05196	3.5	2467
8	0.85	0.0020	0.00493	0.04599	2.7	2469
9	0.95	0.0020	0.00441	0.04125	2.2	2472
10	1.05	0.0020	0.00399	0.03739	1.8	2473
11	1.15	0.0020	0.00364	0.03419	1.5	2475
12	1.25	0.0020	0.00335	0.03150	1.3	2476
13	1.35	0.0020	0.00310	0.02920	1.1	2477
14	1.45	0.0020	0.00289	0.02721	0.9	2478

Star cl. K (N_s=9000000 V_s=0.75 R_s=0.7)

i	a	f_p	p_geom	p_time	n_p	sum
0	0.05	0.0080	0.06513	0.46663	1641.2	1641
1	0.15	0.0040	0.02171	0.18784	110.1	1751
2	0.25	0.0025	0.01303	0.11720	25.8	1777
3	0.35	0.0020	0.00930	0.08514	10.7	1788
4	0.45	0.0020	0.00724	0.06685	6.5	1794
5	0.55	0.0020	0.00592	0.05503	4.4	1799
6	0.65	0.0020	0.00501	0.04676	3.2	1802
7	0.75	0.0020	0.00434	0.04065	2.4	1804
8	0.85	0.0020	0.00383	0.03595	1.9	1806
9	0.95	0.0020	0.00343	0.03222	1.5	1808
10	1.05	0.0020	0.00310	0.02920	1.2	1809
11	1.15	0.0020	0.00283	0.02669	1.0	1810
12	1.25	0.0020	0.00261	0.02458	0.9	1811

Star cl. M (N_s=500000 V_s=0.75 R_s=0.3)

i	a	f_p	p_geom	p_time	n_p	sum
0	0.05	0.0080	0.02791	0.23491	19.7	20
1	0.15	0.0040	0.00930	0.08514	1.2	21
2	0.25	0.0025	0.00558	0.05196	0.3	21

Total for classes F, G, K og M

i	a	f_p	SUM	BIGSUM
0	0.05	0.0080	6907.3	6907
1	0.15	0.0040	501.8	7409
2	0.25	0.0025	119.6	7529
3	0.35	0.0020	49.9	7579
4	0.45	0.0020	30.6	7609
5	0.55	0.0020	20.7	7630
6	0.65	0.0020	14.9	7645
7	0.75	0.0020	11.2	7656
8	0.85	0.0020	8.8	7665
9	0.95	0.0020	7.1	7672
10	1.05	0.0020	5.8	7678
11	1.15	0.0020	4.8	7683
12	1.25	0.0020	4.1	7687
13	1.35	0.0020	2.8	7689
14	1.45	0.0020	2.4	7692
15	1.55	0.0020	1.3	7693
16	1.65	0.0020	1.1	7694
17	1.75	0.0020	1.0	7695
18	1.85	0.0020	0.9	7696

Table 3.2: Number of detected transits for planets with distribution as the currently known exoplanets, for the four star classes and the sum.

Some important traits of the result are: Very few transists are detected around spectral type M. This is because of the small radius of M stars which lowers the geometric and time probabilities, and the relatively small amount of M stars that GAIA will be able to detect, a factor 10 lower than for the other classes, because their luminosities are so small.

Another important trait is that the majority of transits come from planets in close orbits. From 0.05 AU to 0.15 AU that number is already down to one tenth, and it is reduced an additional factor 5 out to 0.25 AU. One would suspect that the number of transits at 0.05 AU was too high because the ditribution of known exoplanets between 0.0 and 0.1 AU is higher towards 0.1 AU. But this is really not the case, the average a in this interval is 0.058.

3.9 Distribution for large a

Planets with large orbital radii also have long periods. The period of Jupiter is for example almost 12 years. Since none of the radial velocity projects have run for that long with the needed precision, such planets have not yet been possible to detect. Jupiter only gives a velocity amplitude of 12.5 m/s, which does not leave much space from the 3-5 m/s which is the typical precision today, and not near at all from the 10-15 m/s which was typical 5 years ago. (The spectrographs of the future, such as HARPS on ESO's 3.6 meter telescope at La Silla, will go down 1 m/s, perhaps the limit where inhomogeneous convection on the star surface drowns the effect of radial velocity changes caused by planets, see [2] 3.2.)

So we can say with certainty that planets with periods of several years are very under-represented in the current data. It is difficult to say if there are any "preferred" planet regions further out, the current distribution does not say anything about it, and theoretical considerations are not much worth without experimental data to test with.

The Solar System, on the other hand, seems to show that giant planets can exist far from their parent star, and that there can be many of them. Current theories of planet formation predict that gas giants should be created far away from the star where different gases like water can condense and add building material to terristrial protoplanets that are big enough to attract the gases of abundant elements like hydrogen and helium. From this perspective the population of 51 Peg-like planets is thought to have migrated inwards from their place of creation several AU out, either by a close encounter with another giant planet, or in the early creation under gravitional influence from the disc of material in the protoplanetary disc, [2] sect. 4.5.

I have therefore tried to run the program from 2.25 AU and out, in jumps of 0.25 AU and with a distribution of 0.03 planets per 0.25 AU (in the Solar System, Jupiter and Saturn give around 1 planet per 5 AU which is 0.05 planets per 0.25 AU, when smoothed out continuously). As usually the program stops running a star class when the increase of planets per interval is less than one.

Star cl. F (N_s=6000000 V_s=0.75 R_s=1.3)

i	a	f_p	p_geom	p_time	n_p	sum
0	2.25	0.0300	0.00269	0.02535	9.2	9
1	2.50	0.0300	0.00242	0.02285	7.5	17
2	2.75	0.0300	0.00220	0.02079	6.2	23
3	3.00	0.0300	0.00202	0.01907	5.2	28
4	3.25	0.0300	0.00186	0.01762	4.4	32
5	3.50	0.0300	0.00173	0.01637	3.8	36
6	3.75	0.0300	0.00161	0.01529	3.3	40
7	4.00	0.0300	0.00151	0.01434	2.9	43
8	4.25	0.0300	0.00142	0.01350	2.6	45
9	4.50	0.0300	0.00134	0.01276	2.3	47
10	4.75	0.0300	0.00127	0.01209	2.1	50
11	5.00	0.0300	0.00121	0.01149	1.9	51
12	5.25	0.0300	0.00115	0.01094	1.7	53
13	5.50	0.0300	0.00110	0.01045	1.6	55
14	5.75	0.0300	0.00105	0.01000	1.4	56
15	6.00	0.0300	0.00101	0.00958	1.3	57
16	6.25	0.0300	0.00097	0.00920	1.2	59
17	6.50	0.0300	0.00093	0.00885	1.1	60
18	6.75	0.0300	0.00090	0.00852	1.0	61
19	7.00	0.0300	0.00086	0.00822	1.0	62

Star cl. G (N_s=8000000 V_s=0.75 R_s=0.9)

i	a	f_p	p_geom	p_time	n_p	sum
0	2.25	0.0300	0.00186	0.01762	5.9	6
1	2.50	0.0300	0.00167	0.01587	4.8	11
2	2.75	0.0300	0.00152	0.01444	4.0	15
3	3.00	0.0300	0.00140	0.01324	3.3	18
4	3.25	0.0300	0.00129	0.01223	2.8	21
5	3.50	0.0300	0.00120	0.01136	2.4	23
6	3.75	0.0300	0.00112	0.01061	2.1	25
7	4.00	0.0300	0.00105	0.00995	1.9	27
8	4.25	0.0300	0.00099	0.00937	1.7	29
9	4.50	0.0300	0.00093	0.00885	1.5	30
10	4.75	0.0300	0.00088	0.00838	1.3	32
11	5.00	0.0300	0.00084	0.00797	1.2	33
12	5.25	0.0300	0.00080	0.00759	1.1	34
13	5.50	0.0300	0.00076	0.00724	1.0	35

Star cl. K (N_s=9000000 V_s=0.75 R_s=0.7)

i	a	f_p	p_geom	p_time	n_p	sum
0	2.25	0.0300	0.00145	0.01373	4.0	4
1	2.50	0.0300	0.00130	0.01237	3.3	7
2	2.75	0.0300	0.00118	0.01125	2.7	10
3	3.00	0.0300	0.00109	0.01031	2.3	12
4	3.25	0.0300	0.00100	0.00953	1.9	14
5	3.50	0.0300	0.00093	0.00885	1.7	16
6	3.75	0.0300	0.00087	0.00826	1.5	17
7	4.00	0.0300	0.00081	0.00775	1.3	19
8	4.25	0.0300	0.00077	0.00729	1.1	20
9	4.50	0.0300	0.00072	0.00689	1.0	21
10	4.75	0.0300	0.00069	0.00653	0.9	22

Star cl. M (N_s=500000 V_s=0.75 R_s=0.3)

i	a	f_p	p_geom	p_time	n_p	sum
0	2.25	0.0300	0.00062	0.00591	0.0	0

Total for classes F, G, K og M

i	a	f_p	SUM	BIGSUM
0	2.25	0.0300	19.2	19
1	2.50	0.0300	15.5	35
2	2.75	0.0300	12.8	48
3	3.00	0.0300	10.8	58
4	3.25	0.0300	9.2	67
5	3.50	0.0300	7.9	75
6	3.75	0.0300	6.9	82
7	4.00	0.0300	6.1	88
8	4.25	0.0300	5.4	94
9	4.50	0.0300	4.8	99
10	4.75	0.0300	4.3	103
11	5.00	0.0300	3.1	106
12	5.25	0.0300	2.8	109
13	5.50	0.0300	2.5	111
14	5.75	0.0300	1.4	113
15	6.00	0.0300	1.3	114
16	6.25	0.0300	1.2	115
17	6.50	0.0300	1.1	116
18	6.75	0.0300	1.0	117
19	7.00	0.0300	1.0	118

Table 3.3: Number of detected planetary transits from planets beyond 2 AU, for the four star classes and the sum.

We see again that M stars don't contribute much, for large orbital radii it's even reduced to nothing at all. Even with a planet frequency several times higher than for the close planets, the outer planets can't compete at all when it comes to transits. The planet frequency in the model is by the way constant and can be taken outside the sum in (3.1). So the numbers in Table 3.3 scale with frequency, that is a twice as great frequency will lead to twice as many transits (in average, the numbers are so small that the dispersion will be very big) in Table 3.3.

3.10 Similar investigations

A task similar to the one presented in this project has been undertaken by N. Robichon in [24]. In this work a planet is considered detected only after a series of observations during transit. Table 3.4 gives the estimated number of transits for two different planet radii and for two different requirements for the number of observations needed for a planet to be detected. A mission life time of 4 and 5 years has been considered. Robichon writes that a photometric follow-up would be very time-consuming, and therefore requires that transits are observed enough times as to determine the period.

Duration of the mission	4 years	5 years
$R_p = 1.0R_{Jup}$, N_{pts}/transit > 5	9400	13700
$R_p = 1.3R_{Jup}$, N_{pts}/transit > 5	25500	36300
$R_p = 1.0R_{Jup}$, N_{pts}/transit > 7	3700	6900
$R_p = 1.3R_{Jup}$, N_{pts}/transit > 7	10100	17800

Table 3.4: Number of transits estimated in [24]

In [25], Erik Høg considers Jupiter-size planets at Earth-like distances from stars. Assuming that a fraction $F = 0.01$ of stars are sufficiently constant and have planets which could give a dimming of 0.01 mag and at the same time have an orbital radius of about 0.5 AU, this leads to about 250 eclipses.

The major differences between the numbers achieved in the two mentioned investigations and the one undertaken in this project lie in the different models applied and the great uncertainty in the distribution of planets. Only a dramatic increase in the number of detected exoplanets of all orbital radii will allow a more exact investigation.

Conclusion

In this project I have discussed strengths and weaknesses in three methods for detecting exoplanets. Radial velocity measurements can find planets in close orbits around their stars, but give no information about the inclination angle, and therefore only the minimum mass of the planet can be established. Astrometry is fit for detecting long-period planets, but require very precise measurements and long time spans. Planetary transits only occur in systems with the proper alignment relative to Earth, but a transit reveals the planet's radius, if the inclination angle can be determined.

	F	G	K	M	Sum
$0 < a < 2$ AU:	3386	2478	1811	21	7696
$a > 2$ AU:	62	35	22	0	118

Table 4.1: Predicted number of transits for the four star classes, for small and large orbital radii, and the sum.

GAIA will detect thousands of transits. Table 4.1 (a summary of Table 3.3 and Table 3.2) gives the predicted number of planets detected from at least one photometric transit detection, with a signal-to-noise ratio of at least 10. The assumed planetary distribution for small orbital radius ($0 < a < 2$ AU) and large orbital radius ($a > 2$ AU) was discussed in Chapter 3. The number of detected planets is, of course, highly sensitive to the assumed planetary distribution, but with the distribution of already detected exoplanets it is now possible to give a qualified guess at the actual distribution for small radius planets. For large a ($a > 2$ AU), the assumed distribution must necessarily still be a guess, and can only be based on the planets of the Solar System and considering theories of planetary formation. The fraction of millimagnitude-constant stars was assumed to be 75 % for spectral classes later than F, and 0 % for earlier stars.

As for the physical and orbital parameters of the planets GAIA will detect photometrically, astrometry is only good for determining inclination angle and mass of the outermost of these planets; a very little part of the total number according to Table 4.1.

The advantage of radial velocity measurements of transit-giving planets (in close orbits, where radial velocity measurements work best) is that $\sin i$ equals 1, no matter if the inclination angle is 85° or 90°. In this way transits give knowledge of the transiting planet's mass, if one can supplement with radial velocity measurements. Unfortunately the planet's radius is very sensitive to inclination angle, and a precise determination of inclination angle requires extra photometry (or more more exact astrometry) from Earth or from space.

So it seems that only a combination of photometry, astrometry and radial velocity measurements can give the enormous catalogue of exoplanets needed to test planetary formation theories, with orbital radius, period, mass, radius and density for each of them. With its observations, GAIA will make a huge contribution to this work.

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Appendix

```
#!/usr/bin/perl

###
###Program used for calculations in planetary transit model
###
###Author: Anders Johansen, 2001
###

###Program starts here

@class = (F, G, K, M); ###Data on star classes
%rad_star = ( F => 1.3, G => 0.9, K => 0.7, M => 0.3);
%num_star = ( F => 6*10**6, G => 8*10**6, K => 9*10**6, M => 5*10**5);
%var_star = ( F => 0.75, G => 0.75, K => 0.75, M => 0.75);

###Data on a intervals and corresponding planetary frequencies
###for close and far domains

@a_close = (0.05,0.15,0.25);
@f_p_close = (0.008,0.004,0.0025);
$delta_a_far = 0.1;
$f_p_far = 0.002;

###File to write data to. Is kept open until end of program.
###Overwrites file if exists.

open FILE, "> /b/98/johan/public_html/bp/model/planet.txt";

foreach $cl(@class)   ###Here starts the sum over star classes
{
$n_s = $num_star{$cl}; ###Get rid of annoying indexes
$v_s = $var_star{$cl};  ###on star class parameters.
$R_s = $rad_star{$cl};  ###All three are reusable dummy variables.

###Header for star class

print FILE " Star cl. $cl (N_s=$n_s V_s=$v_s R_s=$R_s) \n";
print FILE " i   a    f_p   p_geom  p_time   n_p   sum\n";
print FILE "-------------------------------------------\n";

$i = 0; ###Index variable
$n_p = 0; ###Reset number of planets pr. a interval
```

```
$sum = 0; ###Reset sum of planets

while($n_p ge 1 or $i eq 0)  ###Loop stops when too few planets
{
if($i > $#a_close){$a = $a + $delta_a_far;
    $f_p = $f_p_far;}   ###Far domain as specified above.
else{$a = $a_close[$i];
     $f_p = $f_p_close[$i]} ###Close domain.

###Probabilities of transit. R_s in solar radius and a in AU.

$pgeom = (696000*$R_s)/(149600000*$a);
$ptime = 1-(1-($R_s*696000/149600000)/(3.1415*$a))**30;

$n_p = $pgeom*$ptime*$f_p*$n_s*$v_s; ###No. of pl. from model
$sum = $sum + $n_p; ###Sum over intervals

$bigsum_np[$i] = $bigsum_np[$i] + $n_p; ###Sum over classes in
###a intervals.

###Numbers printed to FILE with formats specified.

printf FILE ("%2d",$i);
print FILE " ";
printf FILE ("%3.2f",$a);
print FILE " ";
printf FILE ("%.4f",$f_p);
print FILE " ";
printf FILE ("%.5f",$pgeom);
print FILE " ";
printf FILE ("%.5f",$ptime);
print FILE " ";
printf FILE ("%6.1f",$n_p);
print FILE " ";
printf FILE ("%4.0f",$sum);
print FILE "\n";
$i++; ###Index variable increases.
} ###End of planet class loop.

print FILE "\n"; ###Typographical blank line between star classes.

} ###End of star class loop.

###Next come the number of planets summed over all star classes.

$i = 0; ###Reset for recycling :o).

print FILE "Total for classes F, G, K og M\n";
###Header for table.
print FILE " i   a   f_p    SUM   BIGSUM\n";
print FILE "----------------------------\n";

foreach(@bigsum_np) ###Loop over relevant intervals in a.
{
```

```
        if($i > $#a_close){$a = $a + $delta_a_far;
                    $f_p = $f_p_far;} ###Just like before.
        else{$a = $a_close[$i];
             $f_p = $f_p_close[$i]}

###Acummulated sum over a intervals and star classes.

$reallybigsum = $reallybigsum + $bigsum_np[$i];

printf FILE ("%2d",$i);
print FILE " ";
printf FILE ("%3.2f",$a);
print FILE " ";
printf FILE ("%.4f",$f_p);
print FILE " ";
printf FILE ("%6.1f",$bigsum_np[$i]);
print FILE " ";
printf FILE ("%4.0f",$reallybigsum);
print FILE "\n";

$i++;
} ###End of loop.

close FILE ###Program ends here.
```