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Astrometric accuracy during the past 2000 years

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ABSTRACT: A documentation of the great development of astrometric accuracy since the observations by Hipparchus about 150 B.C. is provided. The development has often been displayed in diagrams, showing the accuracy versus time. These diagrams are discussed, and very significant differences are found, most recently in a diagram from 2007. The diagrams used for Hipparcos up to 1989 are based on a serious misunderstanding of a diagram from 1983. A more correct diagram was constructed in 1995 which was used in the Hipparcos book of 1997. A further improved version is presented here, showing the accuracy of positions and parallaxes in catalogues as based on the included documented data.

Introduction

The present report, including diagrams in an appendix, shall document and discuss the accuracy of observed positions of stars. The evolution of astrometric observations during the past centuries is shown in three tables of a report (Høg 2008b) to which I will refer: Table 1 for position catalogues, Table 2 for proper motion catalogues, and Table 3 for catalogues of trigonometric parallaxes.

The evolution has often been displayed in diagrams, showing the accuracy versus time. These diagrams have at least one thing in common, the improvement by many powers of ten from the one degree of Hipparchus, the Greek father of astronomy, to one milliarcsec for the diagrams including the Hipparcos Catalogue. But Tycho Brahe and Flamsteed are the only other sources always included, though with quite different numbers. Other differences are pointed out below.

I will present a recommended diagram of astrometric accuracy, including explanations and a list of the sources, in literature or otherwise, for the points as they are plotted.

A detailed history of the various other diagrams will be given. Some of the diagrams give the impression of a smooth, gradual improvement over all the centuries, including the last 500 years. This obscures the historically interesting fact that four jumps can be clearly seen in Fig. 1. A 'jump' means a big improvement within a very short time as the result of great investment of material resources and intellectual efforts. First, the Landgrave of Hesse measured positions ten times more accurately than Hipparchus/Ptolemy and Ulugh Beg, but I know too little about the Landgrave to say more. Tycho Brahe was six times more accurate than the Landgrave thanks to an investment never seen before in the history of science. Third, the Hipparcos satellite gave a factor 100 over the contemporary accuracy of positions obtained from the ground. Fourth, the Gaia satellite mission is expected to yield a factor 100 over Hipparcos.

Recommended diagram

Figure 1 is a diagram of the development of astrometric accuracy with time, prepared in 2008 for the present report and for Høg (2008d). The diagram is called Høg-2008 since, for convenience, a diagram is designated by "name-year".

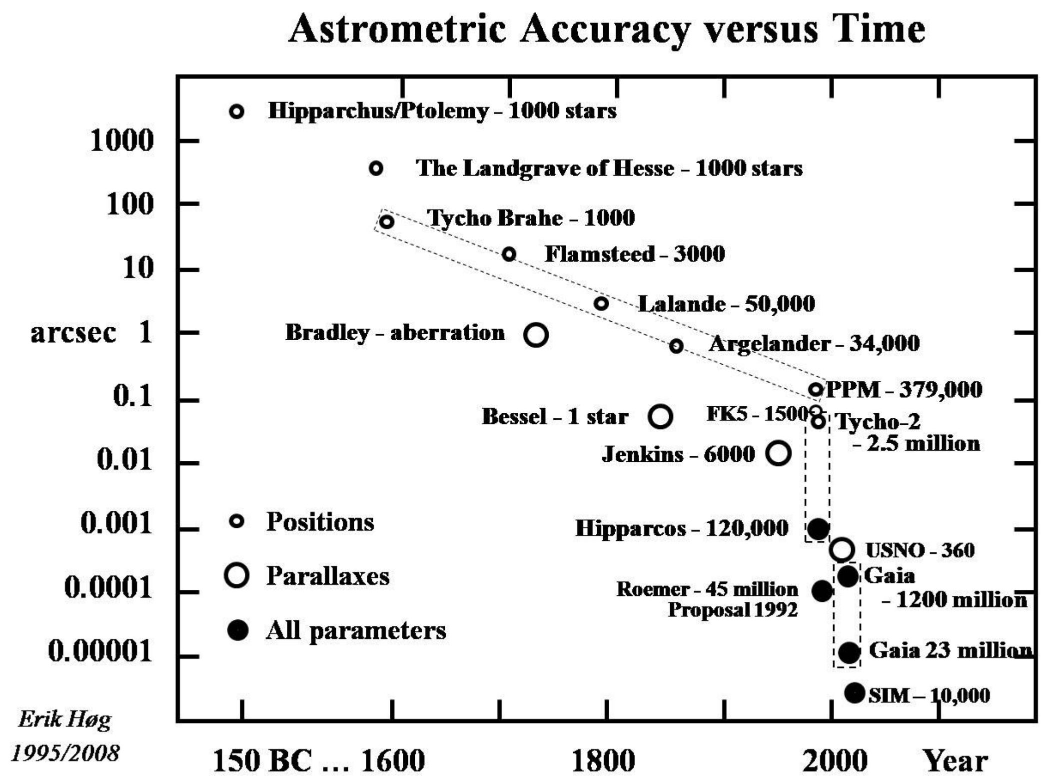

Fig. 1a. Astrometric accuracy during 2000 years: Høg-2008. The accuracy was greatly improved shortly before 1600 by Tycho Brahe. The following 400 years brought even larger but much more gradual improvement before space techniques with the Hipparcos satellite started a new era of astrometry

The first version of this diagram is shown in the appendix as Høg-1995. It was drawn in 1995 in correspondence with several colleagues from the Hipparcos Science Team and appears as Fig.1 in Vol.1 of The Hipparcos and Tycho Catalogues. Two principles were followed in this diagram, but apparently not always in the other diagrams: it shows catalogue errors of single stars rather than errors of single observations and it only shows some of the most accurate catalogues of the given time. To be precise: I am plotting the *median external standard error per star in the catalogues, if available*. In most catalogues bright stars are more accurate than faint ones, but since only one number can be accommodated in the diagram, I find a median value most representative which then corresponds to *the error near the faint end of a catalogue.*

Changes in the diagram compared with Høg-1995/2005 are: Hipparchus/Ptolemy 60' instead of Hipparchus 20', The Landgrave of Hesse is the correct English name instead of Hessen, Flamsteed 20" instead of 12", and 3000 stars instead of 4000, Lalande is now included, for Argelander a larger catalogue of 34000 stars at 0.9", PPM, FK5 and Tycho-2 slightly corrected, Roemer proposal 1992 is included because this proposal led to Gaia and the other astrometry satellite projects DIVA, FAME, and JASMINE. Gaia is here plotted with 1200 instead of "many" million stars, and Gaia is shown with two dots in order to give more information. Bradley-aberration is included, USNO updated to 360 stars instead of 100; the dot for SIM has been placed 3 muas with 10,000 stars, although 1300 stars would be more correct at this accuracy, but space in the diagram is limited; see further explanations in the following section on sources.

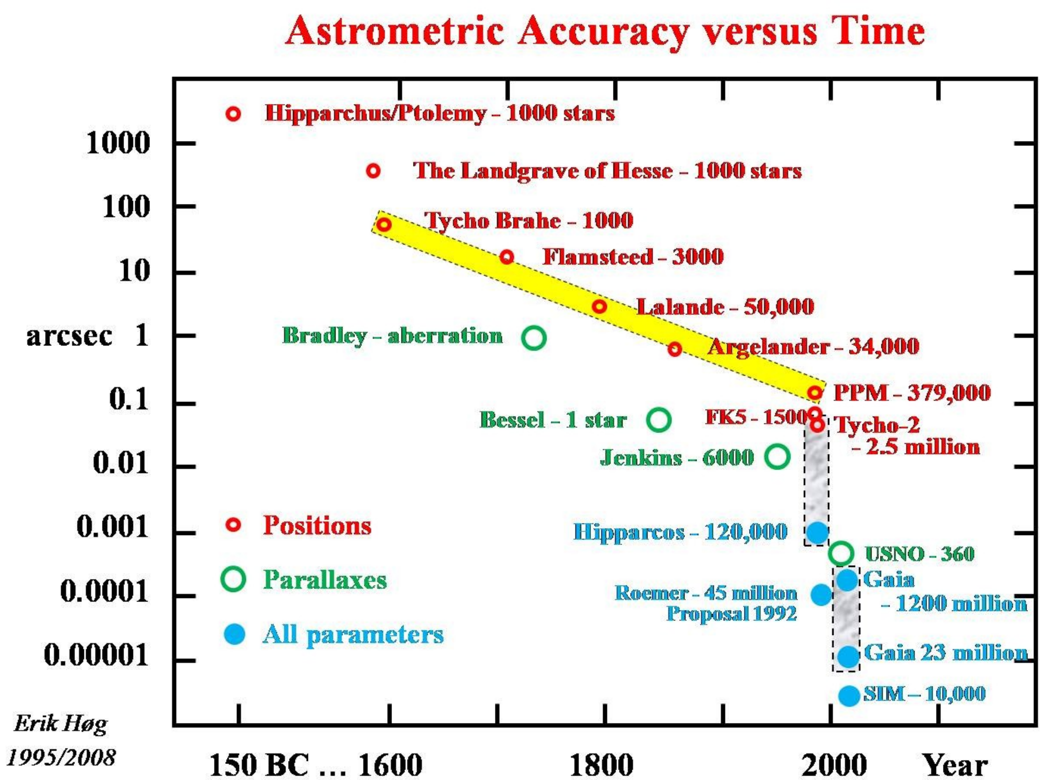

Figure 1b. Astrometric accuracy during 2000 years, Høg-2008. Colour version of Fig. 1a

The points are placed at the mean observation epoch, except the compilation catalogues FK5, PPM, and Jenkins which are placed at the year of publication and with the accuracy of the positions in FK5 and PPM in that year. The circles refer to "positions" and "parallaxes", the word "best" from the previous diagram has been omitted as being misleading because we want to show median values of the standard errors in each catalogue, representative for the bulk of stars in a catalogue. It has been suggested to include more information on the most accurate stars in each catalogue, but the diagram would be more complicated and it would be very difficult to collect the information and to present it well in a graph.

Explanation to the diagram Høg-2008

Brief explanation
Errors of star position coordinates and parallaxes in accurate catalogues are shown in Fig. 1. Tycho Brahe achieved a jump in accuracy of positions through the first "big science" in history.

After four centuries with gradual improvements another much larger jump in accuracy was obtained by the ESA satellite giving the Hipparcos and Tycho-2 catalogues containing a total of 2.5 million stars.

Detailed explanation

Errors of star position coordinates and parallaxes in accurate catalogues are shown Fig. 1. This means the *median external standard error per star in a catalogue, if available*. In most catalogues bright stars are more accurate than faint ones The representative median error, dominated by faint stars, is given for most catalogues.

It appears that the Landgrave of Hesse was able to measure positions with errors about six minutes of arc, ten times better than Hipparchus/Ptolemy in the Antique. A few years after the Landgrave and thanks to generous support from the king of Denmark, Frederik II, Tycho Brahe reduced the errors by a further factor of six. The Landgrave and Tycho, both wanted to equal Hipparchus by reaching the same number of 1000 stars. A period of 400 years followed with gradual improvement of the accuracy as astronomers always made use of the best technical possibilities of their time, especially with better time-keeping equipment and accurate manufacturing of mechanics, optics, and with electronics. The accuracy was improved by a factor about 250 in 400 years, i.e. a factor four per century, and the number of stars was greatly increased.

The introduction of space techniques, however, with the Hipparcos mission gave a veritable jump in accuracy by a factor of 100 with respect to FK5, the most accurate ground-based catalogue ever. Hipparcos obtained a median accuracy of 0.001 arcsec for positions, proper motions and parallaxes of 120 thousand stars. The positions even in the Tycho-2 Catalogue with 2.5 million stars are as accurate as the positions in FK5 containing only 1500 bright stars. Tycho-2 includes annual proper motions, derived from Tycho-2 positions and more than 140 ground-based position catalogues, but no parallaxes. The median standard error for positions of all stars in Tycho-2 is 60 mas, and it is 7 mas for stars brighter than 9 mag. The median error of all proper motions is 2.5 mas/yr.

The points marked “parallaxes” might be labelled “small-angle astrometry” or “relative astrometry”, and all ground-based measurements of parallaxes are of that kind. This is about ten times more accurate than large-angle astrometry which was required to measure the positions shown in the diagram. The first such point is “Bradley – aberration” shown at 1.0 arcsec, the accuracy which Bradley obtained for the constant of aberration with his zenith sector. The accuracy of ground-based parallaxes begins with Bessel’s single star in 1838, followed by a factor 100 improvement in accuracy at the U.S. Naval Observatory in Flagstaff since about 1990 for faint stars.

“All parameters” means that about the same accuracy is obtained for annual proper motions, positions and parallaxes, as was in fact achieved with Hipparcos, for the first time in the history of astronomy. The Roemer proposal of 1992 (Høg 1993) introduced CCDs in integrating scanning mode in a space mission, instead of photoelectric detectors as in Hipparcos. Roemer promised a factor 10 better accuracy than Hipparcos for many more stars, and a development began which led to the Gaia mission due for launch in 2011. For Gaia an improvement by a factor of 100 over Hipparcos is predicted for the 23 million stars brighter than 14 mag, i.e. 10 microarcsec median error. The median accuracy is expected to be 180 microarcsec for the 1200 million stars in the

Gaia catalogue brighter than 20 mag, much better than the accuracy of Hipparcos. The two dots for Gaia thus show the expected accuracy for bright and faint stars. Finally, in view of the expected Gaia results, studies are due about the scientific goals for ground-based optical astrometry after Gaia.

Sources for astrometric accuracy

Here follow the sources and reasoning for the accuracies used in the diagram Høg-2008 of astrometric accuracy, and for Tables 1, 2 and 3 in Høg (2008b) "Selected Astrometric Catalogues", where the references are found, if they are not included in the present report.

The standard errors

Internal errors of observations are obtained by analysis of repeated observations of the same stars at different times, as is usually done in meridian observation catalogues, e.g. in case of USNO (1920) from the n=10 observations. I have then derived the error for Table 1 by division with sqrt(10) because nothing else is available, but this "internal catalogue error" is not given in the catalogue, and it is certainly too small because of the unknown systematic errors.

The three tables should ideally contain the "external errors" of a catalogue entry as would be obtained from a comparison with a more accurate catalogue. Such comparison could be carried out with any of the older catalogues using the now available Hipparcos and Tycho-2 catalogues, if anyone should wish to do so. This has been done for FK5 by Mignard & Froeschlé (2000), and I have used this comparison to derive below that the errors given in the FK5 of positions at the mean epoch and of the proper motions should be multiplied by a factor about 1.6 to obtain external errors. For any other historical catalogue it would be sufficient to take a representative sample of less than a hundred stars for a comparison, but even that would be no small task. Most interesting would be the following catalogues where the errors in Table 1 may be wrong by a factor two: First priority has Wilhelm of Hesse, Flamsteed, Lalande, and Argelander; please inform me if any such study already exists. A thorough comparison of Bradley/Auwers with Hipparcos by Brosche & Schwan (2007) is mentioned below.

In some cases reliable external errors have been derived, e.g. for the Hipparcos and Tycho-2 catalogues in the publications, and for the parallaxes in Jenkins' catalogue by Hertzsprung (1952). In case of Perth70 it is also believed that a reliable external error is known, as explained below at Perth70. The distinction between external and internal errors of catalogues is important for detailed comparisons, but it is difficult in many cases, if at all possible, to find sufficient information about this matter, and it cannot easily be presented in one line of a table. Internal errors are sometimes placed in brackets.

The standard errors in the tables are sufficient for the original purpose, to show the pace of development of astrometric accuracy over very long periods of time. But much care is needed in comparing within short intervals. I have below in some detail compared four meridian circle catalogues from within one century, i.e. two USNO catalogues from observations around 1907 and 1945 are compared with each other, with the Perth70 catalogue observed about 1970, and with the CMC1-11 catalogues observed about 1991.

It appears that the progress in accuracy and efficiency of meridian circles is rather modest in the first half of the 20th century where visual techniques were used, but the progress is very large in the second half thanks to photoelectric techniques and automatic control of micrometer and telescope. This large progress is independent of the Hipparcos mission, but the further progress thanks to the Tycho-2 Catalogue and recording with CCDs is truly tremendous.

The “accuracy of catalogued star positions” is the title of section 3.2.4 in Eichhorn (1974). He discusses theory and practice of this matter in the past where the available means of computation called for simple methods, and in his own time where electronic computers had made rigorous numerical methods feasible. In section 2.2.8 Eichhorn discusses “the accidental accuracy of relative visual positions”. He includes three tables adapted from Cohn (1907b), not (1970) as a typo has produced. Some trivial mistakes both in Eichhorn's extract and in Cohn's original paper make the use of the tables cumbersome, as I discuss below at Bradley/Auwers. This is meant as a call for caution.

Sources for the diagram

The accuracy of an observation catalogue of positions is plotted at the mean epoch, while the catalogues FK5 and PPM, compiled from observations with many instruments, are plotted at the year of publication. The Jenkins compilation of parallaxes is also plotted at the year of publication.

Hipparchus/Ptolemy 1 degree at 150 B.C.
Ulugh Beg also obtained 1 degree accuracy in 1437, but he is not represented in the diagram. The catalogue in the Almagest by Ptolemy is the oldest extant star catalogue. It has been proposed that this catalogue is identical with that of Hipparchus, but this is not supported by Shevchenko (1990). The catalogues of Ptolemy and Ulugh Beg are nearly equivalent in merit, according to Shevchenko. They both have overall systematic longitude errors about one degree, and the systematic error has a scatter about one degree. The root-mean-square errors of the positions of the zodiacal stars in the two catalogues are about 20 arcminutes=1200”=0.33 deg, i.e. within constellations. Shevchenko explains the analogies as due to the fact that the Samarkand astronomers used the equipment and methods described in Almagest.

Eichhorn (1974) p. 101, says that the rms. errors of ecliptic latitudes and longitudes in Ptolemy's catalogue are 0.58 and 0.37 degrees, respectively, but I will stick to Shevchenko.

For the diagrams we have hitherto always shown Hipparchus with 1200”. This is really a local internal error within constellations and I aim at plotting the median external standard error per star which would be 1 degree, and the name should be Ptolemy, not Hipparchus. I have changed the value to 3600” and the name to Hipparchus/Ptolemy; it would be too sad to omit Hipparchus' name entirely.
Very recently, 27 August 2008, F. Mignard informed me of an unpublished study made in 2001 where he compares Ptolemy's catalogue with Hipparcos data. He finds a standard deviation of 0.5 degree using a robust estimator. The following discussion by mail between Mignard and Arenou shows that some issues deserve a closer study. For the time being I will stick to the one degree error, according to Shevchenko (1990), published recently in a refereed journal.

Landgrave of Hesse 360” about 1570
It appears that Tycho Brahe's a little older colleague, Wilhelm IV, called The Wise, Landgrave of Hesse-Kassel (1532-92) was able to measure positions much better than Hipparchus/Ptolemy in the Antique. Eichhorn p. 101 gives an rms error of 6' for the catalogue of 1004 stars, published in 1594 by Wilhelm and Christof Rothmann.

Tycho Brahe 60” at 1586
The accuracy of Tycho Brahe's instruments has been studied by Wesley (1978). For the best of Tycho's nine fundamental stars, he finds an accuracy of 25” for individual measurements with some of the six instruments he considered. He says: “For the majority of the stars that appear in Tycho's final catalogue the overall accuracy may be much less; for there were fewer measurements taken with them...”. I adopt 60” as still plausible for the median standard error.

Flamsteed 20” at 1700
Eichhorn gives an rms. error for Flamsteed of 2”, which must be a misprint for 12” since that is what some others assume. Chapman (1983) cites Schuckburgh and Pearson (respectively 1793 and 1819) for an error of 10”-12”, here is probably where many others took the values.

Other values are quoted by Nielsen (1968). He quotes Argelander (1822) for finding an internal mean error about 7” and an external about 60”. He quotes Piazzi (1813) for a long statement which I condense to: an external mean error of 30” and individual errors exceeding 60”. This together, I settle on 20” for the catalogue which differs a lot from the conventional 12”, but I cannot avoid it.

Lalande 3”
3” from Mineur-1939, 3” from Turon-2007. Arenou (2008) confirms the 3” and calculates the mean epoch to 1795. Lindhagen in 1849AN.....28..129L derives that the number of different stars in Lalande's catalogue is perhaps 40,000, much smaller than the number of entries in the catalogue of about 50,000. The accuracy of 3” can only be valid for the best part of the positions in the catalogue, which is known to contain many errors. F. Mignard notes in a recent mail: “... the Histoire Celeste is a very valuable and extensive description of the sky around 1800 (celebrated as such for example by Olbers), but of low interest in term of astrometric quality. ... In short it is the equivalent in the early 1980 of the SAOC compared to FK4 or GC. ... Histoire Celeste is an astronomical landmark for sideral astronomy, but not for astrometry.”

Argelander 0.9” at 1856
Eichhorn p. 147, gives a mean error of 0.9” for Argelander's large catalogue of 33811 stars from 1867. On p. 143 Eichhorn explains that he assumes that two observations were always combined to give the published position. In the first versions of my diagram I took Argelander's catalogue of 26425 stars from 1844 for which the error is given as 1.1”. I think it is more appropriate to take the larger catalogue, but it makes no significant difference for the diagram.

FK5 62 mas plotted at 1988
The catalogue FK5 states on p. 8 an average “mean error” of individual positions at the mean epoch about 23 mas and of proper motions 0.75 mas/yr. This implies an individual standard error in 1991 of 38 mas, but the error is in fact 62 mas, or 1.6 times larger, as may be concluded from a study by Mignard & Froeschlé (2000) who have compared FK5 with Hipparcos. Their tables 3 and 4 show the local systematic differences, averaged over 230 square degrees, between

Hipparcos and FK5 positions at the Hipparcos epoch of 1991.25. From the tables we find an rms value of 58 mas. Adding the 23 mas gives 62 mas which we consider to be a reasonable estimate of the individual standard error in 1991 and which is therefore adopted for the last column in Table 2.

We tentatively assume that the above factor 1.6 should be applied to the errors on p. 8 giving 40 mas instead of 23 for the error of positions at the mean epoch which is then adopted for FK5 in Table 1. The individual proper motion error becomes 1.2 mas/yr instead of 0.75 and this is adopted in Table 2.

PPM 144 mas plotted at 1992

For Table 2 the standard errors of positions and proper motions are adopted for north and south as given in the catalogue, volumes 1 and 3. This combines to 144 mas for positions for the whole catalogue. It is essential to include PPM in the diagram because it is the last large purely ground-based catalogue before the Hipparcos results appeared. It is therefore more fair to take PPM for comparison with the large catalogues based on space observations, rather than to take the FK5 containing only the very few, very best observed bright stars.

Tycho-2 60 mas at 1991

Tycho-2 includes positions and annual proper motions, derived from Tycho-2 positions and more than 140 ground-based position catalogues, but no parallaxes. The median standard error for positions of all stars in Tycho-2 is 60 mas, and for stars brighter than 9 mag it is 7 mas. The median error of proper motions is 2.5 mas/yr.

Hipparcos 1 mas at 1991

Hipparcos obtained the median accuracy of 1 mas for positions, annual proper motions and parallaxes of 120 thousand stars.

Roemer 0.1 mas at 1992

The Roemer space mission of 1992 (Høg 1993) proposed to use CCDs in TDI mode and promised a factor 10 better accuracy than Hipparcos for many more stars, viz. 0.1 mas as median accuracy for the 45 million stars brighter than 15 mag, and an error better than Hipparcos for the 400 million stars brighter than 18 mag. It is included in the diagram because the Roemer idea led to the Gaia mission, and to the studies of DIVA and FAME. The use of CCDs as modulation detectors was proposed by Høg & Lindegren (1993) but this idea was not further pursued after the superiority of CCDs in scanning mode had been realized.

Gaia 10 and 180 microarcsec at 2015, two dots plotted

Table A. Median astrometric accuracy for Gaia as function of magnitude. Courtesy of Jos de Bruijne.

(1)	(2)	(3)	(4)	(5)
G=06.0-13.0	10.200	8	6	4
G=13.0-14.0	12.700	11	8	6
G=14.0-15.0	24.567	17	13	9
G=15.0-16.0	50.340	27	20	13
G=16.0-17.0	94.486	42	32	21
G=17.0-18.0	170.625	67	51	34
G=18.0-19.0	308.589	112	84	56
G=19.0-20.0	562.010	196	147	98

Column (1) G magnitude range.
Column (2) Number of stars in the G magnitude range (unit is million stars); the sum of column (2) is 1233.517 which is the total number of stars used in the Gaia galaxy model (1.2 billion).
Column (3) Median parallax error for all stars up to the faint magnitude of the magnitude range (unit is muas).
Column (4) Median proper-motion error for all stars up to the faint magnitude of the magnitude range (unit is muas per year).
Column (5) Median positional error for all stars up to the faint magnitude of the magnitude range (unit is muas).

Example: "G=17.0-18.0", "column (4) = 51 muas per year" means that the median proper-motion standard error for all stars brighter than G=18 mag (all stars in the range G = 6-18 mag) is 51 muas per year.

The Gaia mission will be launched in 2011 and a factor of 100 over Hipparcos is predicted for the 23 million stars brighter than 14 mag, i.e. 10 microarcsec median error. The median accuracy for parallaxes and annual proper motions of the 1200 million stars in the final Gaia catalogue is expected to be about 180 microarcsec, much better than the accuracy of Hipparcos. This appears from the following Table A, including explanations by J. de Bruijne.

SIM 3 microarcsec

The dot for SIM has been placed at 3 muas with 10,000 stars, although 1300 would be more correct at this accuracy, but space in the diagram is limited. In fact, a dot at 10 muas with 10,000 stars and another dot at 3 muas with 1300 stars would be more correct.

The NASA interferometric mission (Unwin et al. 2008, Shao 2008) is expected to give global astrometry with few microarcsec accuracy after a five year mission down to 20 mag for more than 10,000 stars. Table 7 in Unwin et al. (2008) gives expected performances, especially 4-20 muas for 10,000 stars of -1.4-20 mag in key projects and 3 muas for 1300 stars of 9-10.5 mag in the astrometric grid.

Narrow angle accuracy of 1 microarcsec per 20 minutes integration is predicted for stars of 6-9 mag. The SIM project has passed all milestones in over ten years of design and development, but is not yet an approved mission and the launch will be after 2014-15.

Bradley-aberration 1” at 1728
The points marked “parallaxes” might be labelled “small-angle astrometry” or “relative astrometry”, and all ground-based measurements of parallaxes are of that kind. This is about ten times more accurate than large-angle astrometry required for the stellar positions in the diagram. The first such point is “Bradley – aberration” shown at 1.0 arcsec, the accuracy which Bradley obtained for the constant of aberration with his zenith sector. According to Arenou (2008) using Flamsteed observations (1689-1697) the precision of aberration can be found within 1.1". This information is from F.G.W. Struve, *Ueber Doppelsterne nach den auf der Dorpater Sternwarte mit Fraunhoffers grossem Fernrohre von 1824 bis 1837*, 1837, page 95:
http://books.google.com/books?id=MEMJAAAAIAAJ&pg=RA2-PA95&lpg=RA2-PA95&dq=flamsteed+aberration+1689+1697&source=web&ots=0YS4rHY2eg&sig=1Kh53ZgrXvLb1BGUeLjbLXbYhhc&hl=fr&sa=X&oi=book_result&resnum=1&ct=result

Bessel 60 mas at 1838
The accuracy of ground-based parallaxes begins with Bessel’s single star in 1838. The 60 mas is based on the analysis below for Table 3. Previous diagrams had, e.g., 60 mas in Høg-1995 and 300 mas in Mineur-1939.

Jenkins 15 mas plotted at 1952
This accuracy for the parallaxes in Jenkins' catalogue was derived by Hertzsprung (1952).

USNO 0.6 mas at 2008
At the U.S. Naval Observatory in Flagstaff, relative parallaxes for 357 faint stars has been obtained with a standard error of 0.6 mas, according to W. van Altena/ C. Dahn (2008 priv. comm.).

The above sources are usually NOT repeated below at the three tables!

Sources for Table 1

Hevelius 20”
Eichhorn (1974) does not give a value for the accuracy of Hevelius. Chapman (1983) p. 136, gives the values 15” to 20” with a reference to Schuckburgh and Pearson from respectively 1793 and 1819 which I have not read. But I adopt the value 20” for my Table 1. Chapman in fact plots a value at 25”.

Rømer 4”
Ole Rømer's only surviving observations with meridian circle in 1706, written in the so called Triduum (three nights), were published by Horrebow (1735). They are discussed by Nielsen (1968) where further references are given. On three nights, 250 transits were observed of 88 stars, the Sun, the Moon, and all the planets known at that time, from Mercury to Saturn. Nielsen has determined the errors of a subset of the star positions by comparison with newer observations and finds external errors in RA of 3.4” and in Dec. 4.5”, which I combine to the one number 4”. This seems to agree with a statement by Piazzi (1813), according to Nielsen.

Lacaille 6”
6” from Mineur-1939; unfortunately I know no primary source. See more below under Piazzi.

Bradley/Auwers 1.1”
Turon-2007 shows 2” for Bradley/Bessel. This is in accordance with the following analysis.

Rather than Bessel's version the one by Auwers should be used, thus Bradley/Bessel/Auwers, which has probably been used for the German fundamental catalogues from Auwers' FC to FK5. Bradley's precision was in general 1", if one should believe http://www.flamsteed.info/fasbradley_files/page0002.htm.

Eichhorn's table II-1 on p.66 gives internal errors of a *single observation*, which is not stated by Eichhorn, but it is by Cohn (1907b) on p.269. The errors are 0.16 s and 1.92” for Greenwich in 1755, i.e. Bradley. But table II-3 gives 0.16 s and 1.3” for one observation by Bradley. Using the formulae in the footnote to table II-1 give however 0.18 s = 2.7” for Dec=0 and 1.92” for zenith distance =0. Rounded to 2” for Bradley/Bessel in accordance with Turon2007. The value is for a single Bradley observation, which may apply to the bulk of the 3222 stars in the catalogue. He did probably make many more observations per star for those few hundred used in the German fundamental catalogues.

It is not clear from Cohn (1907b) or Eichhorn whether this accuracy refers to Auwers' reduction of Bradley/Bessel, and this makes a difference. The version Bradley/Bessel/Auwers obtains an increase of weights compared with Bradley/Bessel of the factors 1.75 in RA and 1.4 in Dec, according to Auwers as quoted by Cohn (1907b), p.269. This would lead to 2”/sqrt(1.6)=1.6”. This is an example how difficult it can be to get a half-way reliable standard error for a catalogue position in Table 1.

Very recently, however, I received Brosche & Schwan (2007) from the first author. It contains a direct comparison of Bradley/Auwers and Hipparcos. For 2450 catalogue values out of the 3268 entries the rms values are 1.2” and 1.0” for respectively RA and Dec. This gives 1.1” for my Table 1, in reasonable agreement with the above 1.6”. The weight has then been calculated using for simplicity the N=3222 in the preceding column, although a smaller number would be more correct since only N=2450 were good enough for the comparison.

Piazzi 1.5”
1.5” from Mineur-1939; unfortunately I know no primary source. F. Mignard wrote in a recent mail: “The most interesting report I found [on Lacaille and Piazzi] is by R. Grant (History of physical astronomy (London 1852) in chap. XIX on the Catalogues of fixed stars from Hipparchus to his time. He praised very much Lacaille care in obtaining absolute measurements on few reference stars. Same opinion about Piazzi work in Palermo using again the 36 fundamental stars of Maskelyne before and building himself a fundamental catalogue of 120 stars before forming his catalogue of 7600 stars. Every stars have been observed several times and "this work is justly considered to be one of the most important that has ever been executed by a single individual".”

Küstner 1908, AC, Stoy 1968, SAOC 1965
Standard errors are taken from Eichhorn p.157, p.279, p.162, p.209.

USNO 1920 and USNO 1952, about 0.15 internal errors
Standard errors are taken from the references in Høg (2008b). Only internal errors are given in the publications as derived from the repeated observations of the same star on different night. These internal errors are divided by sqrt(n) for inclusion in Table 1, because no external error is available. The details for these catalogues are as follows.

USNO (1920) gives the typical internal errors of one observation for RA and Dec on p. A79 and A139 as 0.50” and 0.48”, respectively, which combine to 0.49”. The probable errors used by USNO in those year are converted to standard errors by multiplication with 1.50. With n=10 the 0.15” in Table 1 is obtained.

USNO (1952) gives the typical internal errors of one observation for RA and Dec on p. 375 and 377 as 0.32” and 0.45”, respectively, which combine to 0.37” (as average of the weight from each coordinate). With n=6 the 0.15” in Table 1 is obtained.

These two catalogues are based on respectively 45,000 and 31,000 meridian observations, both obtained in eight years in Washington DC around 1907 and 1945. The development in this period improved the internal error of an RA observation from 0.50” to 0.32” while an observation of Dec stayed about 0.46”.

GC 0.15” and 10 mas/yr
According to Eichhorn (1974) p. 204: “... in the General Catalogue the accidental rms. errors of the positions vary strongly from one star to the next. However, at the epoch they are on the average about 0.15” in both coordinates, and rise to an average of at least 0.70” in 1965 because of the uncertainties of the proper motions (Schlesinger and Barney 1939a).”

Since the (mean) epoch for GC is 1900 this implies a standard error of the proper motions in GC of sqrt(0.7^2-0.15^2)/65 = 0.0105”/yr. The value of 10 mas/yr is adopted for Table 2, but is not stated by Eichhorn; it is however in accordance with the error given by Scott (1963). For Table 1 the value 0.15” is adopted.

Perth70 0.15” external error
Standard errors are taken from the reference in Høg (2008b). Internal standard errors of one observation reduced to zenith is 0.17” and 0.27” for RA and Dec, respectively, cf. Eq. 15, and 0.10 mag for the photoelectric photometry in the visual band. External errors have been derived from observations of circumpolar stars, taking asymptotic errors into account. The typical standard errors of a catalogue position for a program star with four observations are accordingly 0.12” and 0.20” in respectively RA and Dec. This combines to an error per coordinate of 0.15”, adopted for the Table 1.

These internal errors of one Perth70 observation obtained about 1970 are about half the size of those in USNO (1952) and 100,000 such observations were obtained in 5 years in Perth, Western Australia, compared with the 31,000 in 8 years in Washington DC. Thus, a considerable progress in meridian observations were achieved in those years using the photoelectric semi-automatic instrument of the Hamburg-Perth Expedition.

The error of a catalogue coordinate is given as 0.15” in both cases, but they cannot be compared directly because the USNO error is an internal error, the Perth70 error is external.

CMC1-11 1999 and CMC14 2005
Information from the web supplemented by correspondence with D. Evans is shown in Table 1 and explained in Høg (2008b). The CMC1-11 catalogues were obtained with a photoelectric slit micrometer, similar to the one used for Perth70, but with automatic control of micrometer and telescope giving a much higher efficiency. Observed in the better seeing on La Palma and during 14 years instead of 5 years for Perth70 the weight of the catalogue is larger by a factor 30. This is the last meridian circle catalogue in the table where large-angle astrometry is performed. The CMC14 is observed with CCDs in drift-scan mode and the reference stars of the Tycho-2 Catalogue are used for the resulting small-angle astrometry.

USNO-B1.0 2002, UCAC2 2003, GSCII 2005
Information from the web supplemented by correspondence with S. Urban.

2MASS
The 2MASS all-sky catalogue was obtained by two highly automatic telescopes with 1.3 m aperture equipped with HgCdTe detectors sensitive in the J,H,K bands (1-2 microns) with a limit of 17 mag in J. An accuracy of 0.5” for positions was expected, in fact 0.08” was achieved according to N. Zacharias.

Sources for Table 2

Auwers' FC and NFK
For lack of better knowledge, the values are estimated, based on FK3 and N30, therefore the question mark after each of the values.

FK3, GC and N30
Scott (1963) gives an overview, including the proper motion errors for FK3, GC, and N30.

FK4 and FK5
The individual proper motion error becomes 1.2 mas/yr for FK5 instead of 0.75, as derived above under FK5. The error given for FK4 is simply set a bit larger, 2 mas/yr, for lack of better knowledge.

SPM3, UCAC2, USNO-B
All data were received from N. Zacharias in October 2008.

More on proper motions from Arenou
Arenou (2008) mentions two important catalogues: “One led to the discovery of the astrometric binaries: I think that Bessel had 38 stars among which 36 zodiacal stars from Bradley as first epoch (1755) or Maskelyne?. Then, I understand that Argelander had proper motions for 560 stars in 1835 (see 1837MNRAS...4...82A) of which he used 390 to confirm the solar motion.”

More on proper motions from Zacharias
“Traditionally proper motions of stars have been determined by comparing absolute positions (on a fundamental system) at different epochs. With the improvement of the photographic technique in the middle of the 20th century it became possible to image distant galaxies in a sufficient

number to determine absolute proper motions field by field with differential, small angle measures of pairs of plates taken many years apart, covering large areas of the sky for galactic dynamics studies (Wright 1950). This lead to the Northern Proper Motion (NPM) program using the Lick 50 cm double-astrograph (Klemola et al. 1987) and its southern counterpart, the SPM, using the Yale / San Juan instrument of similar design (Girard et al. 1998). These plates, spanning an epoch difference of about 25 years were initially measured with slow but accurate PDS machines for selected stars. By the turn of the century all applicable plates were measured with the PMM at the Naval Observatory Flagstaff station to obtain positions of all stars to 18th magnitude. Reductions are still in progress as part of the UCAC3 effort. Even after no photographic emulsions are any longer in use in astrometry, the development of plate measure machines progressed in the late 20th and early 21st century to allow extraction of all astrometric (and photometric) information available in those data materials."

On reference catalogues

The fundamental catalogues, Auwers FC to FK5, contained too bright and too few stars, FK5 only 1535, to serve directly as a reference catalogue for the reduction of photographic plates. Special observing campaigns were therefore organized to provide denser nets of reference stars for the various photographic surveys, e.g., the AGK3R of 21,499 stars was observed with meridian circles in the 1950s while the AGK3 survey of the northern sky was made. Subsequently, a list of 20,495 Southern Reference Stars was defined and these stars were observed in an international collaboration agreed at the IAU Assembly in Moskau 1958. The resulting SRS catalogue combined with the AGK3R was called International Reference Stars (IRS) which was completed in the 1990s.

The more detailed history of the IRS and the larger ACRS, Astrographic Catalog Reference Stars, is told in the recent message from T. Corbin which I have slightly edited.

"The IRS project originated in the 1960's when T. Corbin was asked to derive proper motions for the observed positions being compiled from the AGK3R observing program. This was to allow the AGK3R positions to be brought to the epochs of the individual AGK3 plates. Only meridian circle catalogs were to be used in order to avoid the color and magnitude terms that older astrograph catalogs would introduce. Catalogs that had been observed using screens were employed to extend the FK4 system to fainter magnitudes, and that extension provided the reductions for the other catalogs. The same thing was done for the SRS.

The IRS then resulted from combining the AGK3R and SRS, each reduced to FK5, and, using the same approach for reducing the older catalogs, computing new mean positions and proper motions on the FK5 system. The FK5 Part II was compiled by combining the FK5 based positions and motions for both FK4 Sup stars selected at Heidelberg and IRS selected for the list at USNO.

ACRS grew from a USNO collaboration with P. Herget in the early 1970's to get improved plate constants for the Astrographic Catalog. The Bordeaux zone was selected, and Corbin compiled a more dense catalog for this part of the sky by combining IRS data with astrograph programs. Herget obtained a significant improvement in the plate solutions, and this showed that compiling such a catalog on a global scale for the reductions of all AC zones would be worth the effort.

The ACRS (Astrographic Catalog Reference Stars) is basically an extension of the IRS.

Particular attention was given to minimizing the systematics in order that the 320,211 stars would represent the FK5 system at the CdC epochs. The PPM was being compiled at Heidelberg at about the same time. PPM includes the AC data, and this is the main difference between it and the ACRS. Both catalogs are based on IRS.

S. Urban used the ACRS database, in combination with Tycho-1 to create a new version of ACRS that then gave an improved set of results for the AC zones. This was all combined to produce the ACT catalog which was quickly superseded by a new version of the proper motions using Tycho-2 results. These were combined with the Tycho-2 observed positions to give the final Tycho-2 Catalogue.

IRS contains 36,027 stars, 124 catalogs were used
errors of proper motions - 4.3 mas/yr in RA and 4.4 mas/yr in DEC
errors of positions - 0.22 arcsec in both coordinates

ACRS contains 320,211 stars, 170 catalogs were used
errors of proper motions - 4.7 mas/yr in RA and 4.6 mas/yr in DEC
errors of positions - 0.23 arcsec in both coordinates at 2000
“

Sources for Table 3

The three first parallaxes
This is here at first retold after Stephen Webb (1999) p.71, and then after F.W. Bessel (1838 and 1840), in both cases abbreviated, followed by my conclusions about the standard errors of the three values as adopted for Table 3.

Quoting Webb (1999): The parallaxes were:
Bessel 0.31” for 61 Cygni (modern value from Hipparcos: 0.287”)
Henderson 1.26” for alfa Cen (Hipparcos : 0.742”)
Struve 0.2619” for Vega (Hipparcos: 0.129”).
(Webb gives the same modern values for the first two stars, but 0.125 for Vega!)

Struve studied Vega with a wire micrometer on the big refractor in Dorpat. Struve made 17 observations during 1836 which gave a parallax of 0.125” with an uncertainty of 0.05”. This was published in 1837. He promised to make more observations and published in 1840 the results of 96 observations made up to 1838. The parallax he obtained this time was 0.2619, more than twice the original result, which cast doubt on both values.

Bessel, meanwhile, studied 61 Cygni with a Fraunhofer Heliometer in Königsberg, using two nearby companions. He began observations in September 1834, but this was interrupted by other work. He returned to the task in 1837 and made 16 or more observations every clear night. As result of his analysis at the end of 1838 he announced a parallax of 0.31” with an error of 0.02”.

Henderson studied alfa Cen with a mural circle from Cape. He completed his observations in 1833, and analysed them upon his return to Scotland later that year. He arrived at a parallax of 1.16” with an error 0.11”. Before publishing his results, however, he asked a colleague to check his work. In the end he published several weeks after Bessel.

More now from Bessel : Bessel (1838) explains his observations and reductions and gives first the annual parallax derived from the star *a* at 8' distance and from star *b* at 12'. They are respectively 0.3690” +-0.0283” and 0.2605” +-0.0278”. The combined solution from *a* and *b* gives 0.3136” +-0.0202”.

Knowing today the very accurate modern values for all three stars, considering them to be the true values, we can derive the true residuals. For Bessel (1838) it is O-C= +0.026”, in good accordance with Bessel's mean error of 0.0202”. That would have led to 20 mas for Table 3, but recently I learnt (Arenou 2008) that two years later, Bessel (1840) gives the value 0.3483” with the mean error 0.0141”, at 0.061” or more that 4 sigma from the true value. I therefore finally adopt 60 mas for the Table 3, also because Bessel's final value will have been the most trusted at his time. In previous diagrams are found 60 mas in Høg-1995 and 300 mas in Mineur-1939.

Struve and Henderson: For Struve's final value O-C= 0.2619”-0.129”=0.133”. This is our best estimate of his standard error, and this estimate has a relative standard error of 1/sqrt(2f)=0.71 since there is f=1 degree of freedom. I adopt 100 mas for Table 3.

Henderson's value gives O-C=1.26”-0.742”=0.518”, much larger than his own claimed error of 0.11”. I adopt the error of 500 mas for Table 3.

Review and catalogue by Oudemans in 1889
I quote Mignard from a mail in Aug. 2008: “I came across the attached reference of interest for your current investigations. This compilation of parallaxes was mentioned in the 'Traite d'Astronomie Stellaire' of Ch. André published in 1899. This is given by him as the Catalogue of the known stellar parallaxes. An interesting point is that in 1899, the analysis of a large number (55) of determinations for 61 Cyg led to pi= 0"44.” - end of quote from Mignard. This catalogue by Oudemans, see Mignard (2008b), is dated 1889, and the “best” value for 61 Cyg was 0.40”, if I read from Tabelle II, i.e. 0.11” too large.

Bigourdan 50/30 mas and Russell 40 mas
The values for both are placed in brackets because they are internal, formal errors. A catalogue by Bigourdan (1909) lists trigonometric parallaxes for about 300 stars, a few with up to 40 observations. The consistency of multiple observations indicates a precision about 50 mas per observation, and a median precision of 30 mas may be inferred for the about 200 stars having more than one observation. Many observations are shown (by bold face) to be the average of several measurements by the same observer, including most of the 100 with only one observation.

The catalogue by Bigourdan is very complete for its time, and may be of interest for further analysis. It is made available in a file, collected by Mignard (2008a).

Russell (1910) presents 52 new photographic parallaxes and claims a standard error about 40 mas.

Schlesinger 15 mas
This is my estimate, based on the value for Jenkins.

Jenkins 15 mas
The 15 mas are from Hertzsprung (1952). It is perhaps interesting to note that this catalogue from 1952 containing photographic parallaxes of 5800 stars has nearly the same accuracy as claimed in 1840 (see above) by Bessel for 61 Cygni, with heliometer. But of course, Bessel observed only one star, with utmost care and with an excellent instrument, and later observations with heliometers gave a much larger parallax. To reach 15 mas and much smaller systematic errors for thousands of stars required an enormous effort in development and implementation. Strand (1963) gives an overview of parallaxes at that time.

Van Altena 10 mas
Bill van Altena has seen the whole Table 3, made no remarks to the rest of it either, and has thus agreed to the information about modern photographic parallaxes.

Hipparcos 1 mas
Hipparcos obtained a median standard error of 1.0 mas for parallaxes.

USNO 0.6 mas and Hubble 0.24 mas
A better accuracy than 1 mas has been achieved from the ground and with the Hubble Space Telescope for several hundred much fainter stars. This informations was received in correspondence with W. van Altena and the informers are named in the table.

Parallaxes according to Westfall (2001)
The numbers of well-measured stars by (year) are about: (1839) 3, (1850) 6, (1862) 10, (1888) 25, (1901) 38. The same source mentions a 1912 catalogue with the parallaxes of 244 stars, determined as follows: 8 with filar micrometers, 83 with meridian transits, 39 by photography, 3 by spectroscopy, and 111 with heliometers.

More on parallaxes from Arenou (2008)
“About the number of parallaxes and the reference by Westfall (2001), one can find that in 1846, Peters has 8 parallaxes (Polaris, Capella, i Ursae maj, Groombridge 1830, Arcturus, Véga, alfa Cygni, 61 Cygni), observations between 1842 and 1843, cf FGW Struve, "Études d'astronomie stellaire", 1847, p 94 (vs Westfall: 1850: 6). In 1889, Oudemans, 1889AN....122..193O, there are 46 stars (vs Westfall: 1888: 25). And then, "The Parallaxes of 3650 Stars of different galactic latitudes, derived from photographic plates", 1908PGro...20....1D, Donner et al.”

Present-day catalogues for astrometric data
A list of presently widely used or well known catalogues for astronomical and especially astrometric data is provided by Zacharias et al. (2004). The list is intended to give users some basic information with regards to the content and usefulness of each. Within each section the catalogues are listed with progressively more and fainter stars but generally with decreasing accuracy.

Other diagrams of accuracy

Here follows a series of diagrams, placed in the sequence I have first seen them. This is the sequence in which the reader can most easily follow the development of the diagram ending with the above Fig. 1. But it is not the sequence in which the diagrams have been published. The first two of the kind were published in 1939 and 1983, but they only came to my knowledge in respectively March and May 2008. Only then did I understand what had made the confusion; these two diagrams by respectively H. Mineur and A. Chapman are shown as Figures 9 and 10.

The diagram Hipparcos-1985 puzzled me in 1985 because a nearly linear development is indicated over 450 years from Copernicus to Hipparcos, even the last piece of 150 years from Simms to Hipparcos fits this line! This cannot be correct, but we had other more urgent tasks in 1985 than to dig deeper here. Four years later the same diagram was used, Hipparcos-1989. In general in these diagrams, one should never draw lines from one point to the next since this indicates that one could interpolate. But it is appropriate to draw a longer line in order to indicate a trend, as has been done in later diagrams, e.g. Høg-1995 and ESA-1998.

Then I saw the diagram Kovalevsky-1990 presented a year later, very different, but again I was puzzled. I wanted to dig deeper, but five years passed before I found the time to make Høg-1995 which was immediately accepted in the Hipparcos Science Team. The jumps in accuracy at Tycho Brahe and at our Hipparcos satellite are clearly seen. Two more versions are shown here as ESA-1998 and Høg-1995/2005.

At the symposium in Shanghai in 2007 Catherine Turon showed the diagram Turon-2007. The smooth curve could give the, I think erroneous, impression that the development had no jumps, but was completely gradual over 550 years from Ulugh Beg to Gaia, though starting to become steeper about 1950.

The diagrams are shown in the sequence they came to my eyes, in the appendix found at:

www.astro.ku.dk/~erik/AccuracyAppendix.pdf

Mignard (mail of August 2008) gives references to further diagrams: "In the book of Walter and Sover (Astrometry of Fundamental Catalogues, Springer, 2000) there is one more diagram of accuracy vs. time on p. 5. The reference is given to: Schmeidler F., 1980, Die Geschichte des FundamentalKataloge, in Astrometrie und Dynamische Astronomie, W. Fricke, Th. Schmidt-Kaler, W. Seggewiss (eds), Mitteilungen der Astron. Gesell. 48, 11-23."

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- Some references are only mentioned in the text, and some are given only in the report Høg(2008b) "Selected Astrometric Catalogues". Here follow other references in the conventional arrangement. Some references, here and in the text, are given merely as the search code to be used at ADS: http://esoads.eso.org/ , e.g. 1840MNRAS...5...55B for Bessel (1840).

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